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Allocation of transmission line fixed charges among joint users according to the benefits

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Allocation of transmission line fixed charges among
joint users according to the benefits

by

Itthi Bijayendrayodhin

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
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DOCTOR OF PHILOSOPHY

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I. INTRODUCTION

A. The Structure of a Power System

A power system consists of a generation system, a transmission system, a subtransmission system and a distribution system. They are roughly equivalent to the manufacture, shipment to market and retailing of any product. The generation system and the transmission system, combined, are referred to as "bulk power supply," and the subtransmission and the distribution systems are the final means to transfer power to the ultimate consumer.

A general characteristic of all electric utility systems is the widespread distribution of electric loads supplied with power from a limited number of electric generators. These generators may be remote from the load areas because of their location at water falls (i.e., hydroelectric plants), near coal supplies or adjacent to large supplies of cooling water. The connection between the loads and generators is through high voltage transmission lines which provide a path for the flow of electric energy. The path, however, is not a single one between a specific generator and a specific load, but is a network of lines with loads supplied at many intermediate points called substations, from which distribution lines may radiate.

Some substations are terminal points for lines which supply power at high transmission voltage and which reduce the voltage to that required in the distribution area. Other substations serve only a

switching function, and provide facilities to permit isolation of portions of the system in the event of trouble and the transfer of power from one line to another.

The removal from service of a faulty line in transmission and subtransmission systems is accomplished by devices called circuit breakers which can switch the lines on and off. The device that determines when a circuit breaker should operate is called a relay. The relays on a power system are complicated devices which continuously measure electric quantities and determine when the power system is in trouble and where the trouble is located.

Transmission lines serve two primary functions. First they carry electric energy from the generators to the loads within a single utility. They also provide paths for electric energy to flow between utilities. These latter lines are called "tie lines" and enable the utilities to operate as a team to obtain benefits which would otherwise not be available.

When power systems are electrically connected by transmission lines they must operate at the same frequency, that is, the same number of cycles per second, and the pulse of the alternating current must be coordinated. As a corollary, generator speeds, which determine frequency, must also be coordinated. The various plants are then said to be operating "in parallel" or "in synchronism" and the system will be said to be "stable." A sharp change in loading at a plant will affect the frequency, but if the plant is strongly interconnected with other plants they will normally help to absorb the effect of the changed loading so

that the change in frequency will be negligible and system stability will be unaffected.

The basic principles of interconnection of electric utility systems date back to the early 1920's. The word "interconnection" (1) means the physical tying in together, by tie lines, of two or more independently owned and managed electric utility systems at their bulk supply levels to form integrated power system or pool.

One of the oldest of such pools is the Pennsylvania-New Jersey-Maryland (PJM) Interconnection, operating in New Jersey and Delaware, in the major portion of Pennsylvania and of Maryland, in a portion of Virginia, and in the District of Columbia. It is a fully integrated pool with coordination of maintenance, operation, planning, and load dispatching.

The Mid-Continent Area Power Planners (MAPP), formed early in 1963, operates in 10 states of Illinois, Iowa, Minnesota, Missouri, Montana, Nebraska, North Dakota, South Dakota, Wisconsin, and Wyoming and in the Canadian province of Manitoba. The maximum demand for electricity of MAPP is expected to reach 30 million kilowatts in 1980.

Other examples of power pools in the United States are the Mid-America Interpool Network (MAIN), the North America Power System Interconnection Committee (NAPSIC), the Canadian U.S. Eastern Interconnection (CANUSE) and so on.

B. Advantages of Interconnection

Interconnections contribute to the two cardinal objectives of power systems operations: (1) economy of power production and (2) continuity

of service.

During normal operating periods, generation is shared. Interchanges between adjacent utilities are scheduled to take advantage of load diversity or available low cost generating capacity, permitting lower overall operating costs and possible deferment of capital investment for new stations. Interconnections provide the ability to use larger plants and relative flexibility in locating them. Scheduled outages for maintenance can be staggered.

During emergencies, spinning reserve capacity is shared, thereby, contributing to continuity of service.

C. Disadvantages of Interconnection

1. The generation control problem (2)

The generating sources of an interconnected system will be spread out over a large area. They will differ in size, type, age, efficiencies and varying response characteristics. There will be buy-and-sell power interchange agreements between adjacent areas. There will be limits to the power that can be carried over certain transmission lines. These are important factors to be considered in generation control.

2. Inadvertent energy interchange (3)

Inadvertent interchange is defined by IEEE standard 94 as the time integral of the net interchange power minus the time integral of the scheduled net interchange.

Growth of system loads and an establishment of new interconnections have resulted in increasing the magnitude of inadvertent energy inter-

change. The inadvertent interchange results from shifts in the net interchange schedule when the frequency is deviated from the standard frequency (60 Hz), power control errors resulting from load variations, from regulating units and from response of telemetering equipments.

3. Cascading outages

In a power pool, a loss of a heavy-loaded transmission line or a loss of a generator in a single utility may cause cascading outages and result in a blackout throughout the power pool. The power failure on November 9, 1965, in the Canadian U.S. Eastern Interconnection (CANUSE) is a good example of cascading outages (4).

D. Responsibility of Interconnections

While sharing in the benefits of interconnected operation, each participant is expected to share comparably in its responsibilities. This involves cooperative participation in system regulation in accordance with the established philosophies of the interconnection, so that smooth, neighborly and mutually beneficial operation is achieved.

E. The Cost of Energy (5)

The major items in the energy cost breakdown are the capital costs, fixed charges, fuel cost and the operation and maintenance costs.

1. Capital costs include all costs necessary to construct a generating plant and prepare it for operation. These costs include engineering, design, interest on borrowed capital, administration, construction and equipment costs.

2. A fixed charge is a constant expense which is paid whether the plant operates or not. Fixed charges include the return on investment, the federal income tax and other taxes, depreciation of the property, insurance and maintenance costs. Fixed charges are usually computed as an annual percentage of the total investment cost, then divided by the plant energy production for a year as a base value, with the answer expressed in mills per kwhr (1 mill = \$0.001).

3. Fuel costs include the material cost (coal, gas), transportation cost or nuclear fuel costs.

4. Operation and maintenance costs are function of both plant capacity and plant operation. They are usually about 10 per cent of the total production costs.

As far as a power system is concerned the capital investment for the generation facilities is about 40 per cent of the total system investment. The distribution system is roughly equal in capital investment to the generation facilities. The transmission and the subtransmission costs are about 20 per cent of the total investment.

There have been transmission problems in setting up power pools since their conception. Transmission lines, including all high voltage lines and tie lines, help everybody, more or less, and all the problems resolve around the question of what your transmission lines do to benefit others and what their lines do for you. Occasionally, jointly-used transmission lines needed for the benefits of a power pool are planned and the difficulty of agreeing on ownership may arise. The benefits each member derives from the line should indicate how much he should

participate in ownership. Each participant should pay fixed charges in proportion to ownership which, in turn, should be proportional to benefits received. It is the purpose of this thesis to introduce an equitable way to allocate fixed costs of a transmission line based on benefits received.

It is presumed in this thesis that benefits of transmission lines consist of a distribution benefit, a wheeling benefit and a reliability benefit. Power flows in the lines indicate a distribution benefit and probabilities of system failure can be used to indicate the reliability benefit of the line. The method used here divides the total fixed charges into two parts, one part assigned to the distribution benefit and the other to reliability. The contribution from each company in the pool is proportional to both benefits rendered. The wheeling benefit is not considered here in detail as it is determined on the basis of contractual agreements.

The thesis is organized as follows. Chapter 2 reviews the literature in this field. The definition of transmission benefits used in the analysis is given in Chapter 3. Chapter 4 discusses sensitivity matrices, both exact and approximate. Together with Appendices B and C it is shown that the sensitivity matrix method could provide quick and easy way to determine quantitatively the distribution benefit defined in Chapter 3.

In Chapter 5 the basic probability theory and the reliability concepts are described. The reliability concept leads to parameters called the failure rate, availability and unavailability of a component

and steady state probability of occurrence.

In Chapter 6 the conditional probability approach is discussed and applied to determine the probability of system failure at each load bus. Quality-of-service criteria have been used to indicate bus failures. The probability of system failure is described in terms of line availabilities and unavailabilities, probabilities of generation outages, and probabilities that the load will exceed the maximum capability of the generators.

In Chapter 7 two methods of allocation are formalized. They are believed to be simple and equitable ways to allocate fixed charges of jointly-used transmission lines. Finally the discussion and conclusions are presented in Chapter 8.

II. REVIEW OF LITERATURE

The research on the problem of seeking a means of allocating fixed costs was divided into two parts. The first part consisted of an examination of the rate structures used in the electric power industry to charge consumers for energy purchased. This study, as will be shown later, could be used to assign fixed costs on a daily or monthly basis as the plant is actually used. It is not well adapted, however, to the planning of future investments. Part A of the Review of Literature discusses methods used in rate making.

The second phase of the research was devoted to a study of methods which might be used to allocate future investments in transmission plant. Part B below reviews the literature which was found to be pertinent in this area.

A. Method of Allocating Demand Costs

The following survey of the literature attempts to reference the important methods of allocating demand costs and to place these in perspective historically. A more detailed discussion of the various methods mentioned is given in Appendix A.

1. The energy method (6, 7)

The simplest and one of the most commonly used methods allocates the demand costs in proportion to the energy used by each class of consumer during a former period, such as a typical month or year. Such a method is simple because the values of energy used by the various

classes during past periods are generally available from records. However it is not fair to all users as it does not account for the cost of providing service which is largely dependent on short time power demands rather than energy.

2. The peak responsibility method (8)

This method allocates the demand costs in proportion to the demand made by each class of consumer on the system at the time of the system maximum demand. The method is an attempt to place the burden upon those classes of consumers responsible for the large amount of investment required to serve the peak load periods, but ignores the energy consumed as a factor.

3. The maximum demand method (8)

The criticism of the peak responsibility method suggested that the demand costs may be more equitably allocated by the ratio of the maximum demand of the class under consideration to the summation of the maximum demands of all classes regardless of time of occurrence with respect to one another or with respect to the system peak. Again this method neglects the energy required by each class of customer.

4. The Greene's method (9)

This method uses a combination of the maximum demand and the energy methods. Part of the demand cost is a direct function of the maximum demands and the remainder is a direct function of energy consumed. The proper values can be obtained by solving two simple equations.

5. The Eisenmenger theory (10)

Eisenmenger made a most elaborate study of central station load curves and their relative contribution to the demand costs of the system. He advocated a simplified method of allocation. Eisenmenger's method will be found more equitable than the first three previous methods, because it takes into consideration not only the so-called on-peak but also the off-peak load of the various consumer classes and their durations.

6. The phantom method (11)

Hills asserted that a fair and just division of cost will be on an energy basis. With a plant operating at 100% load factor, the demand costs divided by the number of kilowatt-hours generated and multiplied by the consumption of each customer at the generating plant will give the true demand costs that should be allocated to each customer.

In actual practice, the load factor is usually not 100%. But the line of reasoning will still apply if account is taken of an imaginary customer called the "phantom customer" needed to give the ideal condition.

7. The weighted peak method

Reed (12), in 1927, in an effort to correct some of the defects of Greene's (9), Eisenmenger's (10), and Hills' (11) methods in overcharging the off-peak customers, presented a new method which was called "the weighted peak method." This method allocates the demand costs to the various classes of consumer according to the share of each class in the total weighted peak. The weighted peak of any class of business is taken as equal to the demand of that class at the time of the plant

peak plus a fraction of the difference between the maximum demand of that class of business and its demand at the time of the plant peak. This fraction that is added is the ratio of the plant demand at the time of the class maximum demand as compared with the total peak demand.

All the methods of allocating demand costs are discussed in greater detail in Appendix A.

B. Methods Useful in Planning Future Investments

In a conventional pooling arrangement the economic benefits of shared generation reserve and savings from energy interchange are realized. Of course, all the participating systems must attain a position of equity with respect to the responsibility for overall generating reserves and savings. Determination of this equity can be made only if the tools are available for measuring the relative contributions and benefits in projected and actual operation, and only if an equitable method of allocating the benefits is developed. Many methods have been developed for use in power pools.

In 1950, Watchorn (13) described a way to determine capacity benefits resulting from an interconnection of generating systems and used this to justify the installation of transmission facilities as a substitute for generating capacity. Also, he gave several possible bases for allocating such benefits. He pointed out that when only two systems are involved, in an interconnection, the resulting capacity benefit should be divided equally between them. But when more than two systems are involved, the benefit allocated to any one of the participating systems should not be reduced by the addition of any new participants into the interconnection. He suggested what may be termed "the mutual

benefits method of allocation" which recognizes that the benefit should be divided among the participating systems in proportion to the benefits for all combinations of two's among them. The method suggested meets the two basic requirements so long as the installed capacity requirements are determined on the basis of consistent application of probability methods.

In 1957 Phillips (14) illustrated a method which he reasoned to be more equitable to allocate saving from energy interchange in power pools where more than three companies are involved. He pointed out that it is a generally accepted principle throughout the United States that on interchange where only two parties are involved, the savings are divided equally between buyer and seller. The accounting involved in applying this theory is given by a simple equation for the billing rate which is a function of energy interchange, replacement cost of purchasing company and supplying cost of selling company. When the magnitude of the interconnection grows to include three companies, the accounting is slightly more complicated since, for any specified period, either one company is buying and two are selling, or two companies are buying and one is selling. Again, the total interchange can be broken down into two separate 2-party transactions and no arbitrary method is involved for determining the distribution of energy. A similar equation for billing rate can be applied in the 3-company interconnection. When the magnitude of the power pool grows to four companies, it is no longer possible to say which company receives a given block of power except in those hours when only one company is buying or only one company is

selling. He suggested that if more than one company is buying during the particular period, each buying company's replacement cost is compared with the weighted average of all the selling companies in order to determine the billing rate. Conversely, in any period the selling cost of any selling company is compared with the weighted average of the replacement costs of all the buying companies for that specific period in order to determine the billing rate for that company.

It is very difficult to determine an equitable method of allocating the fixed charges of the interconnection facilities for power interchange among the various participants. Suggestions have been made that such fixed charges be divided annually among participating parties of the interconnection arrangement on the basis of the actual dollar benefits derived by the individual members from power interchange transactions. Watchorn (13) recommended that such allocation may well be on approximately the same basis as the allocation of the capacity benefits. Bary, in his discussion in (14), asserts, however, that the disposition of fixed charges on interconnection facilities should be made at the time they enter into an interconnection agreement. He further suggests that benefits should be allocated on an equitable basis with the amounts applicable to each participating system to remain fixed for a prolonged period, and be subjected to modification only as a result of future changes in the scope or extent of the facilities involved in the interconnection, or due to major changes in the components of fixed charges (comprising return, taxes, depreciation, insurance, and maintenance). He argued that the disposition of fixed charges should

not be made automatically dependent upon the actual day-to-day or year-to-year operational benefits of power interchanges.

Anthony (15) described the exchange of seasonal diversity capacity between Tennessee Valley Authority (TVA) and the South Central Electric Companies (SCEC). Basically, each SCEC company was to own, operate and maintain those Extra High Voltage (EHV) facilities required in its "service area." Financing was to be handled on a group basis. The annual cost of ownership, operation and maintenance of individual company facilities was to be prorated to each company by an arbitrary formula based on: 1) the portion of such facilities installed by that company compared with the total EHV facilities installed by all SCEC companies, and 2) the percentage participation by that company in diversity capacity exchange of TVA power. Since the company in whose service area EHV facilities are installed is in a position to use the facilities for purposes other than the interchange of power with TVA, each company owning EHV facilities was to begin to absorb 5% of the annual charges of those EHV facilities in its service area. Each year thereafter, for a total of 10 years, the amount to be absorbed was to be increased by 5%. Consequently, at the end of the 10-year period, annual charges to be shared by the companies was projected to be 50% of the initial annual charges. Incremental losses occasioned by the receipt or delivery of power under the agreements were to be distributed in proportion to each company's participation in each power transfer.

Firestone, et al., (16) presented an extension in the use of probability techniques for analyzing a system's generation reserve

position and applied this method to the Central Area Power Coordination Group (CAPCO) system. A probabilistic capacity model is merged with a load model to develop the expected frequency distribution of daily capacity margins. The daily capacity margin is considered to be the difference between the load that exists during a daily peak period and the operable capacity at that time. Operable capacity for this purpose is the normal rating of installed generating capacity, adjusted for various limitations, plus purchases of firm power from other utilities, less outages both planned and forced. Each of these capacity margins is, of course, associated with the probability of the corresponding capacity level.

The CAPCO group, like other power pools, required a mechanism for insuring the equitable sharing of benefits and responsibilities arising from such an association. The fundamental basis of equity adopted by the CAPCO group was that each party should contribute to the group reserve in the same proportion as he expected to utilize it. Negative margins were quite useful as the measure of a system's need for help from outside the pool, whereas the positive margins were used as the measure of a system's ability to provide help to outside systems. An energy quantity called "megawatt-days" was developed as a useful measurement here. "Positive megawatt-days" are equal to the sum of the products of each positive margin and its respective frequency. "Negative megawatt-days" are calculated in a similar manner, from the negative margin data. By proper distribution of capacity responsibility it is possible to make the relationship of each party's contribution to the

group reserve (positive megawatt-day value) to his potential use of the group reserve (negative megawatt-day value) equal to that for each of the other parties. The capacity responsibility assigned represents the power in megawatts for which the individual party bears financial responsibility.

A 1967 paper by Rincliffe (17) describes the Pennsylvania-New Jersey-Maryland (PJM) policy for allocating the annual costs of the 500 KV transmission system to all pool members. The 500 KV transmission system, owned by six companies, was being constructed to bring power from the mine-mouth stations to the load centers and to provide high capacity interpool tie lines. The total cost of the transmission system was divided into an inter-area tie function and a generation delivery function. The inter-area function was allocated to all PJM members and associated systems in proportion to their sizes as measured by peak loads. The generation delivery function was allocated to the owners of the stations in proportion to ownership of the combined capacity of these stations.

So far, the methods described have realized the benefits of interconnection facilities from the savings, to the participating systems, due to power interchange transactions. But Brandt, in his paper (18), defined the benefits to be gained from transmission facilities in a more comprehensive way. Those benefits are personal company benefits which include distribution benefits and wheeling benefits, and pool benefits or a value associated with increased reliability of the pool. These benefits have recently been examined by a number of researchers.

De Sieno and Stine (19) discuss the mechanics of component failure and repair, and show that the power system behavior follows a Markov process. The reliability of simple system configurations can be evaluated analytically by solving the Markov equations. However for more complex systems a digital computer simulation is recommended because of the large number of distinguishable system states.

A frequency and duration method dealing with different system configurations was the subject of two papers by Gaver et al. (20) in 1964 and Montmeat et al. (21) in 1965. Both outage duration and outage frequency are predicted by making certain specific assumptions regarding the probability distributions of component repair and failure times. An important aspect of this approach is the introduction of a varying environmental condition associated with the operating component. Two states, normal weather and stormy weather, are used to describe the component environment. Each condition has an associated component failure rate in terms of failures per year of operation within that environment and an approximate expression for the overall outage rate is used. Another important aspect of the approach is that it considers circuit overloads under outage contingencies on a probability basis. The probability of overloads are obtained by sampling the annual load curves of the transmission lines for contingencies of different durations. A digital computer program has been developed to apply the reliability calculation technique to actual power system networks.

Mallard and Thomas (22) illustrated an application of the method described in Reference 20 to analyze the reliability of a transmission

system including the effect of interconnections and generation. The network configuration, equipment performance characteristics and system operating conditions were included in the analysis through the use of a load contingency curve. Approximately 300 load flow runs were required to determine the necessary information for the relative simple system used.

Billinton and Bollinger (23) discuss the basic concepts of stationary Markov processes and particularly their application to transmission system reliability evaluation. Transmission components were assumed to operate within a 2-state fluctuating environment described by normal and stormy weather conditions. Markov processes were used to determine the probabilities of failure for simple configurations. The paper shows that certain system components failure rates are not greatly affected by the environment and therefore do not require the complete 2-state condition.

Billinton (24) describes a method of evaluating the reliability at any point in a composite system including both generation and transmission facilities. The method involves a conditional probability approach. This approach is later applied to a practical configuration utilizing a computer program (25, 26, 27), using a service quality standard as the reliability criterion rather than simple continuity between sources and load points.

Bhavaraju (28) illustrates, by a study of a simple system, the conditional probability method as described in (24) and (25) and discusses some practical aspects of the method. The conditional

probability approach will be discussed in greater detail later.

In this thesis, a new approach to the analysis of the benefits of a jointly-used transmission line is introduced. This approach is based on the idea of transmission benefits described in (18). The distribution benefit will be clearly defined and be realized quantitatively from power flows in the line. The wheeling benefit will not be considered in this thesis in detail as it is determined on the basis of contractual agreements. The reliability benefit of the line to the system will be realized quantitatively from probabilities of system failure. Then an arbitrary combination of the benefits will be assumed to formalize a method to allocate the fixed charges of the jointly-used transmission line.

III. DEFINITION OF BENEFITS

The benefits to be gained from a transmission line are of two types, viz., personal company benefits and pool benefits. The personal company benefits include the distribution benefit and the wheeling benefit. The pool benefit to be obtained from the transmission line is attributable to the reliability of service. The wheeling benefit will not be defined and will not be considered in this thesis (consult Reference 18).

A. Definition of Distribution Benefit

The distribution benefit of a transmission line to a power system is defined as the increment of real power flowing over that line when the total load of that power system is changed from one arbitrary load level to a higher arbitrary load level under economic production schedules from the dispatch control center under normal conditions, measured either in the physical units (MW or kW, etc.) or in per unit (pu) on some appropriate base.

In a power pool consisting of many power companies, the distribution benefit defined above can be realized for each company by changing the company load level one company at a time while keeping the other companies load level fixed. As the load is changing from one level to another higher level the magnitude of power flows are changed. It could mean that an increment of real power needed to serve the load changes is distributed to a given line.

It will be assumed in this thesis that an average load level of a power pool is chosen as the lower base load level and the company

load level which occurs at the time of the pool peak is chosen for that company as the higher base load level. The increments of real power flowing over the line can be found by conventional digital load flow analysis or by an approximate method which will be described in Chapter IV.

B. Definition of Pool or Reliability Benefit

The reliability benefit of a transmission line to a power system is defined as an increment of the total probability of system failure, calculated at the load buses in the power system, with the line in service and with the line out of service.

The total probability of system failure in the system is the summation of the probabilities of system failure to serve the load at each load bus in the system. The probabilities are calculated at each of the individual load buses in the power system.

The conditional probability approach will be used to calculate the probability of system failure. The criteria to determine whether a system fails to serve the load at any bus will be described in Chapter VI. Only the voltage criteria will be considered in the analysis. The conventional digital load flow analysis will be used in the calculation also.

IV. SENSITIVITY MATRICES AND DISTRIBUTION BENEFIT

A. Complete ac Power Flow Equations

The power flow equations (29) in an interconnected system of N nodes are described by a set of $2N$ real simultaneous equations:

$$P_k = E_k \sum_{m=1}^N E_m Y_{km} \cos(\theta_k - \theta_m - \phi_{km}) \quad [4.1]$$

$$Q_k = E_k \sum_{m=1}^N E_m Y_{km} \sin(\theta_k - \theta_m - \phi_{km}) \quad [4.2]$$

or

$$P_k = \sum_{\alpha} \frac{E_k E_{\alpha}}{Z_{k\alpha}} \sin(\theta_k - \theta_{\alpha} - \delta_{k\alpha}) + \sum_{\alpha} \frac{E_k^2}{Z_{k\alpha}} \sin \delta_{k\alpha} \quad [4.3]$$

$$Q_k = - \sum_{\alpha} \frac{E_k E_{\alpha}}{Z_{k\alpha}} \cos(\theta_k - \theta_{\alpha} - \delta_{k\alpha}) + \sum_{\alpha} \frac{E_k^2}{Z_{k\alpha}} \cos \delta_{k\alpha} + \frac{E_k^2}{Z_{kg}} \cos \delta_{kg} \quad k = 1, 2, \dots, N \quad [4.4]$$

where

P_k = Real power injected at node k .

Q_k = Reactive power injected at node k .

E_k = Voltage magnitude at node k .

θ_k = Phase angle of the voltage at node k .

Y_{km} = Magnitude of the elements in nodal admittance matrix.

ϕ_{km} = Phase angle of the elements in nodal admittance matrix.

α = Index of a node directly connected with node k .

$\bar{Z}_{k\alpha}$ = Impedance of branch $k_{\alpha} = Z_{k\alpha} \exp [j(\pi/2 - \delta_{k\alpha})]$.

Z_{kg} = Impedance between node k and ground (assume pure reactance).

B. AC Power Flow Equations Considering Real Power Only

The voltages are assumed to be a constant throughout the network due to the action of perfect regulators at each node. Only the real power in Equations 4.1 and 4.3 are retained.

1. Quadratic approximation

This section discusses the second-order Taylor series expansion of the simplified ac power flow equations (the angle differences are generally small enough so as to neglect their third power).

From Equation 4.1

$$\begin{aligned}
 P_k &= E_k \sum_{m=1}^N E_m Y_{km} \cos(\theta_k - \theta_m - \phi_{km}) \\
 &= E_k \sum_{m=1}^N E_m Y_{km} [\cos(\theta_k - \theta_m) \cos \phi_{km} + \sin(\theta_k - \theta_m) \sin \phi_{km}] \\
 &= E_k \sum_{m=1}^N E_m [\cos(\theta_k - \theta_m) Y_{km} \cos \phi_{km} + \sin(\theta_k - \theta_m) Y_{km} \sin \phi_{km}]
 \end{aligned}$$

$$\text{Let } \bar{Y}_{km} = Y_{km} \angle \phi_{km} = G_{km} + j B_{km}$$

$$\therefore G_{km} = Y_{km} \cos \phi_{km}$$

$$\text{and } B_{km} = Y_{km} \sin \phi_{km}$$

$$P_k = E_k \sum_{m=1}^N E_m [G_{km} \cos(\theta_k - \theta_m) + B_{km} \sin(\theta_k - \theta_m)]$$

Expand the cosine and the sine function by the Taylor series and neglect their power terms greater than third power. We have

$$\cos (\theta_k - \theta_m) \approx 1 - \frac{(\theta_k - \theta_m)^2}{2!}$$

$$\sin (\theta_k - \theta_m) \approx (\theta_k - \theta_m)$$

and

$$P_k \approx E_k \sum_{m=1}^N E_m \left[G_{km} - \frac{G_{km}(\theta_k - \theta_m)^2}{2!} + B_{km}(\theta_k - \theta_m) \right] \quad [4.5]$$

From Equation 4.3

$$\begin{aligned} P_k &= \sum_{\alpha} \frac{E_k E_{\alpha}}{Z_{k\alpha}} \sin (\theta_k - \theta_{\alpha} - \delta_{k\alpha}) + \sum_{\alpha} \frac{E_k^2}{Z_{k\alpha}} \sin \delta_{k\alpha} \\ &= \sum_{\alpha} \frac{E_k E_{\alpha}}{Z_{k\alpha}} [\sin(\theta_k - \theta_{\alpha}) \cos \delta_{k\alpha} - \cos(\theta_k - \theta_{\alpha}) \sin \delta_{k\alpha}] \\ &\quad + \sum_{\alpha} \frac{E_k^2}{Z_{k\alpha}} \sin \delta_{k\alpha} \\ &= \sum_{\alpha} \frac{E_k E_{\alpha}}{Z_{k\alpha}^2} [\sin(\theta_k - \theta_{\alpha}) Z_{k\alpha} \cos \delta_{k\alpha} - \cos(\theta_k - \theta_{\alpha}) Z_{k\alpha} \sin \delta_{k\alpha}] \\ &\quad + \sum_{\alpha} \frac{E_k^2}{Z_{k\alpha}} \sin \delta_{k\alpha} \end{aligned}$$

From the definition

$$\begin{aligned} \bar{Z}_{k\alpha} &= Z_{k\alpha} \angle \pi/2 - \delta_{k\alpha} \\ R_{k\alpha} + jX_{k\alpha} &= Z_{k\alpha} [\cos(\frac{\pi}{2} - \delta_{k\alpha}) + j \sin(\frac{\pi}{2} - \delta_{k\alpha})] \\ &= Z_{k\alpha} [\sin \delta_{k\alpha} + j \cos \delta_{k\alpha}] \end{aligned}$$

$$\therefore R_{k\alpha} = Z_{k\alpha} \sin \delta_{k\alpha}$$

and

$$X_{k\alpha} = Z_{k\alpha} \cos \delta_{k\alpha}$$

$$\begin{aligned} \therefore P_k &= \sum_{\alpha} \frac{E_k E_{\alpha}}{Z_{k\alpha}^2} [X_{k\alpha} \sin(\theta_k - \theta_{\alpha}) - R_{k\alpha} \cos(\theta_k - \theta_{\alpha})] \\ &\quad + \sum_{\alpha} \left(\frac{E_k}{Z_{k\alpha}} \right)^2 R_{k\alpha} \end{aligned}$$

By the same approximation

$$\begin{aligned} P_k &\approx \sum_{\alpha} \frac{E_k E_{\alpha}}{Z_{k\alpha}^2} [X_{k\alpha} (\theta_k - \theta_{\alpha}) - R_{k\alpha} \{ 1 - \frac{(\theta_k - \theta_{\alpha})^2}{2!} \}] \\ &\quad + \sum_{\alpha} \frac{E_k^2}{Z_{k\alpha}^2} R_{k\alpha} \\ &\approx \sum_{\alpha} \frac{E_k E_{\alpha}}{Z_{k\alpha}^2} [X_{k\alpha} (\theta_k - \theta_{\alpha}) - R_{k\alpha} + R_{k\alpha} \frac{(\theta_k - \theta_{\alpha})^2}{2!}] \\ &\quad + \sum_{\alpha} \frac{E_k^2}{Z_{k\alpha}^2} R_{k\alpha} \\ &\approx \sum_{\alpha} \frac{E_k E_{\alpha}}{Z_{k\alpha}^2} X_{k\alpha} (\theta_k - \theta_{\alpha}) + \sum_{\alpha} \frac{E_k E_{\alpha}}{Z_{k\alpha}^2} R_{k\alpha} \frac{(\theta_k - \theta_{\alpha})^2}{2!} \\ &\quad - \sum_{\alpha} \frac{E_k E_{\alpha}}{Z_{k\alpha}^2} R_{k\alpha} + \sum_{\alpha} \frac{E_k^2}{Z_{k\alpha}^2} R_{k\alpha} \end{aligned} \quad [4.6]$$

where

$R_{k\alpha}$ = Resistance of branch $k\alpha$.

$X_{k\alpha}$ = Reactance of branch $k\alpha$.

2. Linear approximation (d.c. power flow model)

To make a linear approximation of Equations 4.5 and 4.6 the squared terms in these equations are also neglected, either because the difference is small compared to one or because R is smaller than Z , or both.

$$P_k \approx E_k \sum_{m=1}^N E_m [B_{km} (\theta_k - \theta_m) + G_{km}] \quad [4.7]$$

or

$$P_k \approx \sum_{\alpha} \frac{E_k}{Z_{k\alpha}^2} [E_{\alpha} X_{k\alpha} (\theta_k - \theta_{\alpha}) + E_k R_{k\alpha} - E_{\alpha} R_{k\alpha}]$$

Assume that $E_k \approx E_{\alpha}$, then

$$P_k \approx \sum_{\alpha} \frac{E_k E_{\alpha} X_{k\alpha}}{Z_{k\alpha}^2} (\theta_k - \theta_{\alpha}) \quad k = 1, 2, \dots, N \quad [4.8]$$

C. Jacobian Matrix

The Jacobian matrix (30) gives the linearized relationship between small changes in voltage angle, $\Delta\theta$, and magnitude, ΔE , and small changes in real power, ΔP , and reactive power, ΔQ . Equations 4.1 and 4.2 are derived from the following power equations.

$$P_k + jQ_k = \sum_{m=1}^N \bar{Y}_{km}^* \bar{E}_m^* \bar{E}_k \quad [4.9]$$

$$= \sum_{m=1}^N Y_{km} E_m E_k e^{j(\theta_k - \theta_m - \phi_{km})} \quad [4.10]$$

where

$$\bar{Y}_{km} = Y_{km} \angle \phi_{km} = G_{km} + jB_{km}$$

$$\bar{E}_m = E_m \angle \theta_m = e_m + jf_m$$

$$\bar{E}_k = E_k \angle \theta_k = e_k + jf_k$$

N = number of nodes

By forming the total differentials, the following linear relationships can be found for small variations in the variables of Equation 4.10.

$$\Delta P_k = \sum_{m=1}^N H_{km} \Delta \theta_m + \sum_{m=1}^N N_{km} \Delta E_m \quad [4.11]$$

$$\Delta Q_k = \sum_{m=1}^N J_{km} \Delta \theta_m + \sum_{m=1}^N L_{km} \Delta E_m$$

H_{km} , N_{km} , J_{km} , and L_{km} are coefficients which can be expressed as partial derivatives. The derivation of these coefficients is shown in Appendix A.

For $k \neq m$

$$H_{km} = \partial P_k / \partial \theta_m = a_{mk} f_k - b_{mk} e_k$$

$$N_{km} = \partial P_k / \partial E_m = (a_{mk} e_k + b_{mk} f_k) / E_m$$

$$J_{km} = \partial Q_k / \partial \theta_m = - (a_{mk} e_k + b_{mk} f_k)$$

$$L_{km} = \partial Q_k / \partial E_m = (a_{mk} f_k - b_{mk} e_k) / E_m$$

[4.12]

For $k = m$

$$\begin{aligned}
 H_{kk} &= \partial P_k / \partial \theta_k = -Q_k - B_{kk} E_k^2 \\
 N_{kk} &= \partial P_k / \partial E_k = P_k / E_k + G_{kk} E_k \\
 J_{kk} &= \partial Q_k / \partial \theta_k = P_k - G_{kk} E_k^2 \\
 L_{kk} &= \partial Q_k / \partial E_k = Q_k / E_k - B_{kk} E_k
 \end{aligned} \tag{4.13}$$

where we have arbitrarily defined the current-dimensioned quantity

$$a_m + jb_m = (G_{km} + jB_{km})(e_m + jf_m) . \tag{4.14}$$

Then the total current flowing from node k to all other nodes (except ground) is

$$\begin{aligned}
 a_k + jb_k &= \sum_{m=1}^N (a_m + jb_m) \\
 &= \sum_{m=1}^N (G_{km} + jB_{km})(e_m + jf_m) .
 \end{aligned} \tag{4.15}$$

By definition,

$$\begin{bmatrix} \Delta P_k \\ \Delta Q_k \end{bmatrix} = \begin{bmatrix} \text{Jacobian} \\ \text{Matrix} \end{bmatrix} \begin{bmatrix} \Delta \theta_m \\ \Delta E_m \end{bmatrix} \tag{4.16}$$

where

$$\begin{bmatrix} \text{Jacobian} \\ \text{Matrix} \end{bmatrix} = \begin{bmatrix} H_{km} & N_{km} \\ J_{km} & L_{km} \end{bmatrix} .$$

From Equations 4.1 and 4.2, the small variations in real power and reactive power can be represented in the matrix form

$$\begin{bmatrix} \Delta P_k \\ \Delta Q_k \end{bmatrix} = \begin{bmatrix} H_{km} & N_{km} \\ J_{km} & L_{km} \end{bmatrix} \begin{bmatrix} \Delta \theta_m \\ \Delta E_m \end{bmatrix} \quad [4.17]$$

and

$$\begin{bmatrix} \Delta P_k \\ \Delta Q_k \end{bmatrix} = \begin{bmatrix} H_{km} & N_{km} \\ J_{km} & L_{km} \end{bmatrix} \begin{bmatrix} \Delta \theta_m \\ \Delta E_m \end{bmatrix} \quad [4.18a]$$

D. Approximations to the Jacobian Method

In general, for a small change in the magnitude of bus voltage the real power at the bus does not change appreciably. Likewise, for a small change in the phase angle of the bus voltage, the reactive power does not change appreciably. Therefore, the simplified matrix equation (31) is given by the approximate equation

$$\begin{bmatrix} \Delta P_k \\ \Delta Q_k \end{bmatrix} \approx \begin{bmatrix} H_{km} & 0 \\ 0 & L_{km} \end{bmatrix} \begin{bmatrix} \Delta \theta_m \\ \Delta E_m \end{bmatrix} \quad .$$

If one is interested in only the variation of real power then

$$[\Delta P_k] = [H_{km}][\Delta \theta_m] \quad [4.18b]$$

We shall call this the approximate method.

The elements of the Jacobian Matrix can also be calculated by using rectangular coordinates (31); the approximations could be determined by

neglecting the off-diagonal elements of the submatrix \underline{H}_{km} , \underline{N}_{km} , \underline{J}_{km} , and \underline{L}_{km} .

From Equation 4.5 with quadratic approximation

$$k \neq m$$

$$\partial P_k / \partial \theta_m = E_k E_m [G_{km}(\theta_k - \theta_m) - B_{km}] \quad [4.19]$$

$$k = m$$

$$\partial P_k / \partial \theta_m = E_k \sum_{m=1}^N E_m [B_{km} - G_{km}(\theta_k - \theta_m)] \quad [4.20a]$$

and the approximate power matrix equation becomes

$$\begin{bmatrix} \underline{\Delta P} \end{bmatrix} = \begin{bmatrix} \frac{\partial (P_1, P_2, \dots, P_N)}{\partial (\theta_1, \theta_2, \dots, \theta_N)} \end{bmatrix} \begin{bmatrix} \underline{\Delta \theta} \end{bmatrix} \quad [4.20b]$$

For the dc power flow, the equations given in Equations 4.7 and 4.8 are already linear in θ . But from Equation 4.8 which is the only dc model equation of power flow considered in this thesis

$$\partial P_k / \partial \theta_\alpha = -(E_k E_\alpha / Z_{k\alpha}^2) X_{k\alpha}^2 \quad \text{for any } \alpha \neq k \quad [4.21]$$

$$\partial P_k / \partial \theta_k = \sum_{\alpha} \frac{E_k E_\alpha X_{k\alpha}^2}{Z_{k\alpha}^2} \quad [4.22]$$

and the dc power matrix equation again becomes

$$[\underline{\Delta P}] = [\underline{Y}] [\underline{\Delta \theta}] \quad [4.23]$$

where the off diagonal elements and the diagonal elements of the matrix \underline{Y} are shown in Equations 4.21 and 4.22, respectively.

E. Sensitivity in Power Systems (32)

Sensitivity is defined as the ratio $\Delta x/\Delta y$ relating small changes Δx of some dependent variable to small changes Δy of some independent or controlled variable y . In power systems, two dominant types of sensitivity relations are defined, namely:

- 1) The sensitivity of electrical variables, such as the voltage E_i and the voltage phase angle θ_i at bus i with respect to other electrical variables, such as real and reactive power, P_k and Q_k at bus k ,
- 2) The sensitivity of the operating cost with respect to such electrical variables as the consumption at node i and production at node j .

In the following discussion, stress will be put on the first type of sensitivity relation. The calculation of this type of sensitivity relation requires the inversion of the Jacobian matrix (either exact or approximate) associated with the power flow equations.

From Equation 4.16 we have, for the Jacobian method

$$\begin{bmatrix} \Delta \underline{\theta}_m \\ \Delta \underline{E}_m \end{bmatrix} = \begin{bmatrix} \text{Jacobian} \\ \text{matrix} \end{bmatrix}^{-1} \begin{bmatrix} \Delta \underline{P}_k \\ \Delta \underline{Q}_k \end{bmatrix} \quad [4.24]$$

From Equation 4.18b we compute, for the approximate method

$$[\Delta \underline{\theta}_m] = [\underline{H}_{km}]^{-1} [\Delta \underline{P}_k] \quad [4.25]$$

Finally from Equation 4.23, we have, for the dc model method

$$[\underline{\Delta\theta}] = [\underline{Y}]^{-1} [\underline{\Delta P}] \quad [4.26]$$

F. Modified Sensitivity Matrix

In solving the network, two quantities of the swing bus, i.e., the voltage magnitude E and the voltage phase angle θ remain unchanged. Because of this fact two of the elements in Equations 4.24, 4.25, and 4.26 are zero and the sensitivity matrices in these equations should be modified. With the assumption that the voltages at all regulated buses remain unchanged, the modified sensitivity matrices can be further modified. The new sets of dependent variables and of independent variables are related by a new matrix equation. These ideas are explained clearly below.

1. In Equation 4.24, $\Delta\theta_1$, the change in the voltage phase angle of the swing bus, is a dependent variable and ΔP_1 , the change in the real power injected at the swing bus, is an independent variable. But at this particular bus $\Delta\theta_1 = 0$ (known value) and ΔP_1 is unknown. Thus the two quantities should be interchanged. As a result we have ΔP_1 as a dependent variable and $\Delta\theta_1 = 0$ is independent. The same idea applies to $\Delta E_1 = 0$ and ΔQ_1 . The Jacobian then becomes a "hybrid" matrix relating the new sets of dependent and independent variables.

2. All other values of ΔP_m (excluding $m = 1$) are independent variables and these should be given to start the calculation. All other values of $\Delta \theta_m$ ($m \neq 1$) are dependent and these will be found as the result of the calculation.

3. At all regulated buses, the set of ΔE_n (n corresponding to regulated bus numbers) are zero and they are dependent variables in Equation 4.24. Similarly the ΔQ_n (n corresponding to regulated bus number) are independent variables. In the calculation ΔQ_n are a part of the result so they should be dependent variables. Again, ΔE_n and ΔQ_n should be interchanged.

4. At all other buses, except the swing and the regulated buses, ΔQ_p are independent variables and they are the data used to start the calculation. They should remain at the place where they were in Equation 4.24. Similarly for the corresponding set of voltages ΔE_p .

From the above explanation we have a new matrix equation 4.27.

$$\begin{bmatrix} \Delta P_1 \\ \hline \Delta \theta_m \\ \hline \Delta Q_1 \\ \hline \Delta Q_n \\ \hline \Delta E_p \end{bmatrix} = \begin{bmatrix} \beta_1 & \beta_2 \\ \hline \beta_3 & \beta_4 \end{bmatrix} \begin{bmatrix} \Delta \theta_1 = 0 \\ \hline \Delta P_m \\ \hline \Delta E_1 = 0 \\ \hline \Delta E_n = 0 \\ \hline \Delta Q_p \end{bmatrix} \quad [4.27]$$

where

$\Delta P_1, \Delta Q_1$ = Change in the real and reactive power
injected at bus 1 (swing bus).

$\Delta \theta_1, \Delta E_1$ = Change in the voltage phase angles and
magnitude at bus 1 (swing bus) = 0, 0.

m = Bus nos. ($\neq 1$).

n = Regulated bus nos. only ($\neq 1$).

p = Other bus nos. (except 1 and n).

B_1, B_2, B_3, B_4 = Partitioned matrices of the modified
sensitivity matrix.

An example of the modified sensitivity matrix from Equation 4.27 is shown in Table C.3. The sensitivity method described by this equation will be referred to as the Jacobian Matrix Inverted method or simply the Jacobian method.

Similarly for the approximate method we compute a new matrix in the form

$$\begin{bmatrix} \Delta P_1 \\ \underline{\Delta \theta}_m \end{bmatrix} = \begin{bmatrix} H_{km}^{-1} \end{bmatrix} \text{ modified } \begin{bmatrix} 0 \\ \underline{\Delta P}_m \end{bmatrix} \quad [4.28]$$

For the dc method we have a similarly modification, namely

$$\begin{bmatrix} \Delta P_1 \\ \underline{\Delta \theta}_m \end{bmatrix} = \begin{bmatrix} \gamma^{-1} \end{bmatrix} \text{ modified } \begin{bmatrix} 0 \\ \underline{\Delta P}_m \end{bmatrix} \quad [4.29]$$

Examples of the modified sensitivity matrices in Equations 4.28 and 4.29 are shown in Table C.4 and C.5. The sensitivity methods described by Equations 4.28 and 4.29 will be referred to as the approximate method and the dc model method respectively.

Any of these sensitivity matrix methods provides a quick and easy way to determine a new state of a power system from small changes in loads at particular buses. The accuracy depends on a base case in which the coefficients of the matrix are evaluated and on the amount of change in the loads. The method is actually the one-iteration Newton-Raphson Method. For planning purposes the sensitivity matrix method should give reasonable results. Another advantage of this method is that, once a base case has been chosen, the matrix can be stored on a magnetic tape for later study. What is left to do is to supply changes in loads at the buses.

Equations 4.27, 4.28, and 4.29 will be applied to a sample system to determine new system states and the additional power flow in each line in the system following small change in load. The results obtained by solving these equations will be compared with the results obtained from using the conventional power flow analysis. The details of these calculations are shown in Appendix C.

As defined in Chapter 3 an additional power flow will be used to indicate the distribution benefits of transmission lines in a power system. Therefore, the sensitivity matrix methods can be used to determine the distribution benefit directly.

G. Per Unit Quantity of the Distribution Benefit

The distribution benefit of a transmission line can be measured in either physical real power units (MW, kW, etc.) or in per unit (p.u.). The benefit in p.u. can be determined in several ways, depending on how the base value is chosen. The following bases are suggested.

1. The base value may be chosen to be any arbitrary number in consistent physical unit throughout the study (as usually done in load flow analysis) such as 100 MW, etc. Then

p.u. distribution benefit of a line or p.u. ΔP line

$$= \frac{\Delta P_{\text{line}} \text{ in MW}}{100 \text{ MW base}} .$$

2. The base value may be chosen to be the total change in area load in MW (ΔP_{area} in MW). Then

p.u. distribution benefit of a line

$$= \frac{\Delta P_{\text{line}} \text{ in MW}}{\Delta P_{\text{area}} \text{ in MW}} .$$

3. The base value may be chosen to be the sum of the changes in every line flow (MW) in the area. Then

p.u. distribution benefit of a line

$$= \frac{\Delta P_{\text{line}} \text{ in MW}}{\Sigma \Delta P \text{ in all lines}} .$$

4. The base value may be chosen for each individual line separately to be the total sum of the distribution benefits of that particular line to each separate area in the pool.

As will be seen in Chapter 7, we need, for a line, a single base value of distribution benefit, that is common to all areas in the system, to determine p.u. distribution benefits of the line to each area separately. The base value should include the effect of all area changes in load. This is the basis of allocating a part of the line fixed charges that contributes to distribution. The base value suggested in methods 2 and 3 don't include the effect of all area changes in load. There is nothing wrong with the base value suggested in method 1 as long as the value chosen is not too high. In this thesis, the base value suggested in method 4 will be used to determine p.u. distribution benefits. The results are shown in Table 4.1. Obviously the base value of distribution benefit may not be the same in any of the four sensitivity methods described in Appendix C.

Table 4.1. Per unit quantity of distribution benefit obtained from the dc model method

	Distribution benefit (MW) to		Base value of Dist. benefit	Distribution benefit (p.u.) to	
	Area 1	Area 2		Area 1	Area 2
ΔP_{1-4}	6.04	4.00	10.04	0.6	0.4
ΔP_{1-6}	7.926	0.615	8.541	0.927	0.073
ΔP_{2-3}	0.0	2.88	2.88	0.0	1.0
ΔP_{2-5}	1.352	2.12	3.472	0.39	0.61
ΔP_{3-4}	1.367	6.7	8.067	0.17	0.83
ΔP_{4-6}	4.64	0.0	4.64	1.0	0.0
ΔP_{5-6}	0.135	2.1	2.235	0.06	0.94

H. Discussion of the Results

From the sensitivity computations (results in Appendix C) the following salient features have been observed.

1. For the sample system, the computer execution time for each method of sensitivity computation is as follows:

conventional load flow method - 2 seconds

the Jacobian method - 1.31 seconds

the approximate method - 0.34 seconds

The calculation of the dc model method was done by a calculator. If the computer program is developed the execution time should be in the same order as the approximate method.

The execution times of the Jacobian method and the approximate method include times to compute the modified sensitivity matrices. If the modified sensitivity matrices are stored on a magnetic tape, they can be called and the total execution times in later studies would be much smaller.

2. The sensitivity methods suggested are not practical for a large system in a sense that the matrix to be inverted will be very large. The order of the matrix will be $2n$ for the Jacobian method and will be n for both the approximate method and the dc model method, where n is the total number of buses. It would be costly to invert such a large matrix if there are, say, 300 buses in the system. Once the matrix has been inverted and stored on a magnetic tape (after a base case has been chosen), the whole calculation is simple and fast.

3. The computer program which is used in the computation is designed to give the magnitude of the increment of line flows only. It does not give the direction of flows as the conventional load flow program does. The program is shown in Appendix D.

4. The accuracy of the sensitivity method depends on the base case chosen to calculate all elements in the matrix and size of the load changes. It is not as accurate as the conventional load flow method but the results seem reasonable for certain purposes such as future planning.

Referring to Table C.9, the increment of line flows in the transformer connecting bus 5 and bus 6 calculated by the conventional method and by the dc model method are quite different. This is a disadvantage of the dc model method. It gives a large percentage error in the branches with small flows or in branches with a flow reversal. As shown in Figure 6.2 the flow in the transformer connecting bus 5 and bus 6 has the magnitude of 0.34 Mw in the direction from bus 6 to bus 5. But at the average load level (Figure C.1) the flow has the magnitude of 1.91 Mw in the direction from bus 5 to bus 6. The dc model method will have trouble with this transformer. But in an actual analysis we are interested in high voltage lines which are usually heavily loaded.

5. The several approximations are reviewed as follows.

a. The Jacobian method, Equation 4.27 needs only one approximation, i.e., the increments of real power and reactive power are small compared with the original real and reactive power (base case). The coefficients in the Jacobian Matrix, Equations 4.11 and 4.12, are derived from the original non-linear equations.

b. The approximate method, Equation 4.28, is based on the following assumptions. First, this method ignores the corresponding variations in reactive power (ΔQ). Second, it assumes that a small change in the magnitude of bus voltage has negligible effect on the corresponding change in the magnitude of real power. The coefficients in the $[H_{km}]$ matrix, Equation 4.18b, are also derived from the original non-linear equations.

c. The dc model method, Equation 4.29, needs an additional approximation. The coefficients in the $[Y]$ matrix, Equation 4.23, are derived from a linearized equation, Equation 4.8, of the original non-linear real power equation only.

The results obtained from these three methods are compared in Table C.6. The results obtained from the Jacobian method are quite comparable with the results from the conventional load flow method. The approximate method and the dc model method give similar results. The dc model method offers no striking advantage in the analysis over the approximate method. The only reason that one might prefer the dc model method is that it is a well-known technique and has been utilized widely in other work (Reference 29).

d. The purpose of this analysis is to introduce a quick and easy way to determine the distribution benefit in MW or in per unit for planning purposes. From the reasons above it is believed that the dc model method is a reasonable technique to use. The method is fast and is believed to be accurate enough for long ranged planning purposes (see discussion of Reference 29).

V. PROBABILITY AND RELIABILITY

A. Basic Probability Theory

The word probability is recognized as a technical word implying "a measure of chance." The probability of an event A is defined as the fraction of the favorable outcomes of the event A to the total number of trials in a test, if each trial has an equal chance to result in the event A. Thus, when each trial ends either with a favorable outcome of the event A or with an unfavorable outcome, which is denoted as event B, and there are X outcomes with the attribute A, and Y outcomes with the attribute B, then the total number of trials is X+Y, and the probability of A is defined as

$$P(A) = \frac{X}{X+Y}$$

Equally, the probability of B will be defined as

$$P(B) = \frac{Y}{X+Y}$$

Strictly speaking, these probabilities are only approximations of the true probabilities P(A) and P(B). Their exact value could be obtained only from an infinite number of trials. But when the fixed number of trials X+Y is reasonably large, the estimate will be close to the true probability.

B. Rules for Combining Probabilities

1. Two events are said to be "independent" if the occurrence of one event does not affect the probability of occurrence of the other event.

2. Two events are said to be "mutually exclusive" if they cannot both happen at the same time (e.g., success and failure).

3. The probability of simultaneous occurrence of two or more independent events is the product of the respective event probabilities.

Using set theory notation $P(A \cap B)$ is the probability of the occurrence of "both A and B." If A and B are independent events:

$$P(A \cap B) = P(A) \cdot P(B) .$$

4. If two or more events are mutually exclusive then the probability of occurrence of any of the events is the sum of the respective event probabilities. Using set theory notation $P(A \cup B)$ is the probability of occurrence of either A or B or both. If A and B are mutually exclusive then they cannot both occur and

$$P(A \cup B) = P(A) + P(B) .$$

5. If two events are independent but not mutually exclusive then the probability of the occurrence of either one or both is given by

$$P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B) .$$

If the two events are mutually exclusive then the probability of their simultaneous occurrence $P(A) \cdot P(B)$ is zero and $P(A \cup B) = P(A) + P(B)$ which agrees with 4 above.

6. When extra conditions are imposed on a certain portion of the event population then the probability associated with the subpopulation events are called conditional probabilities.

7. The probability of the simultaneous occurrence of two events is equal to the product of the probability of the first event and the conditional probability of the second event determined under the assumption that the first event has occurred.

The probability of A given B is usually written as $P(A|B)$ where the vertical bar is read as "given", i.e., the probability of A given that B has occurred. Then the probability of both A and B is written as

$$P(A \cap B) = P(A) \cdot P(B|A)$$

$$P(A \cap B) = P(B) \cdot P(A|B) .$$

If A and B are independent events, then

$$P(B|A) = P(B)$$

$$P(A|B) = P(A) .$$

8. If the occurrence of an event A is dependent upon a number of events B_j which are mutually exclusive then

$$P(A) = \sum_{i=1}^j P(A|B_j) \cdot P(B_j) .$$

If the occurrence of an event A is dependent upon only two mutually exclusive events for component B, success and failure, designated B_x and B_y , respectively, then

$$P(A) = P(A|B_x) \cdot P(B_x) + P(A|B_y) \cdot P(B_y) .$$

The conditional probability approach is very useful and will be discussed later in more detail.

9. Mathematical expectation: consider a probability model with outcomes $x_1, x_2, x_3, \dots, x_n$ and define the probability of occurrence of each as $p_1, p_2, p_3, \dots, p_n$. The expected value of the variable x is defined as

$$\begin{aligned} E(x) &= p_1x_1 + p_2x_2 + p_3x_3 + \dots + p_nx_n \\ &= \sum_{i=1}^n p_i x_i \end{aligned}$$

The expected value is the weighted mean of the possible values, using their probability of occurrence as the weighting factor. It is not something that is "expected" in the ordinary sense but is actually the long term average as the number of trials increase to infinity. It is often called the population mean.

C. Basic Reliability Concepts

When an event A is the survival of a component and an event B is its failure, then using the definition of probability, the reliability of a component can be defined as the fraction of components surviving a test to the total number of components present at the beginning of the test.

1. The general reliability function

When a fixed number N_0 of components are repeatedly tested, there will be, after the time t , N_s components which survive the test and

N_f components which fail. Therefore, $N_o = N_s + N_f$ is a constant throughout the test because as the test proceeds, the number of failed components N_f increases exactly as the number of components surviving decreases. The reliability or probability of survival is defined at any time t during the test to be (33)

$$R(t) = \frac{N_s}{N_o} = \frac{N_s}{N_s + N_f} \quad [5.1]$$

where N_s and N_f are counted at that specific time t . Thus, reliability measured in such a test is a function of the operating time t . We can also define the probability of failure Q (called unreliability) as

$$Q(t) = \frac{N_f}{N_o} = \frac{N_f}{N_s + N_f} . \quad [5.2]$$

At any time t it is apparent that

$$R(t) + Q(t) = 1 .$$

We can show that

$$R(t) = \exp \left[-\int_0^t \lambda(t) dt \right] \quad [5.3]$$

where $\lambda(t)$ is defined as a failure rate of a component. The detail of derivation is given in Appendix E.

When we can specify that λ is constant, the probability of a component surviving a time t is given by

$$R(t) = e^{-\lambda t} . \quad [5.4]$$

The instantaneous failure density function $f(t)$ is defined by

$$f(t) = -dR(t)/dt = \lambda e^{-\lambda t} \quad [5.5]$$

for λ constant.

2. Mean time to failure (34)

If x is a continuous random variable with a probability density function $f(x)$, the expected value $E(x)$ is given by

$$E(x) = \int_0^{\infty} x \cdot f(x) dx \quad [5.6]$$

If λ can be assumed constant, the density function $f(t)$ is given by Equation 5.5. Then

$$E(t) = \int_0^{\infty} \lambda t \cdot e^{-\lambda t} dt \quad [5.7]$$

Integrating by parts, let

$$u = t$$

$$dv = \lambda e^{-\lambda t} dt$$

$$\int u dv = uv - \int v du$$

Then the expected value is found to be

$$E(t) = \left[-te^{-\lambda t} \right]_0^{\infty} - \int_0^{\infty} e^{-\lambda t} dt = \frac{1}{\lambda} \quad [5.8]$$

The expected value is often designated as the Mean-Time-to-Failure and in the useful life period (λ is constant) is the reciprocal of the failure rate. The Mean-Time-to-Failure m is a constant average duration of the successful operating periods of a component, observed and counted for a long time. Similarly, if the duration of each outage is observed and averaged over a large number of failures a constant average outage time r is found. The reciprocal of r is the average rate of repair μ .

Experience has shown that many components follow a relatively standard failure rate pattern with time. This basic pattern sometimes called a "bathtub curve," is shown in Figure 5.1. Region 1 is known as the "burn-in" or "debugging" period and could be due to manufacturing

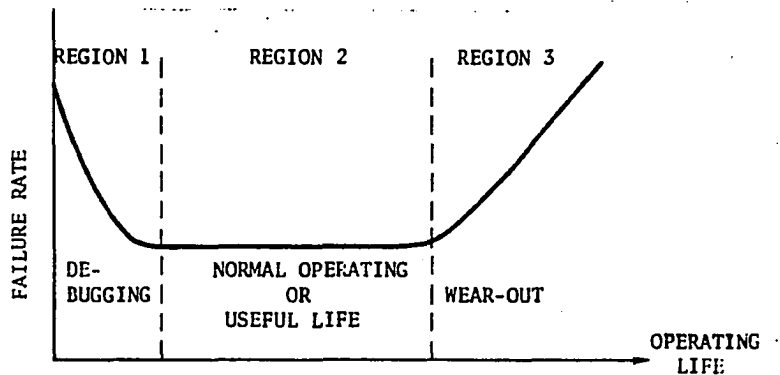


Figure 5.1. Component failure rate as a function of age

errors or improper design. The failure rate decreases comparatively rapidly as those weak components fail one by one. The component population reaches its lowest failure rate level which is approximately constant. This period of life is called the "useful life" period because it is in this period that the components can be utilized to the greatest advantage. Region 3 represents the wearout period and is characterized by a rapidly increasing failure rate with time.

Power system components such as generating units, transformers, switchgear, etc. can remain within the useful life period by constant and careful preventive maintenance. In this way insulation and mechanical elements are not allowed to enter a wearout state before

they are replaced. This is an extremely important assumption, however, as reliability prediction based upon useful life rates is invalid and extremely optimistic if the system contains components which are operating within their wearout period.

Consider the case of a single repairable component for which time to failure and repair time are exponentially distributed and independent of each other. The component can exist in only two states, either available or unavailable (under forced outage). Obviously the component states are independent of each other. The availability of a given state is the mean time in that state divided by the mean cycle time for that state to occur (35). The cycle time T of a component is the sum of the mean time to failure m and the mean repair time r and is known as the Mean-Time-Between-Failures (MTBF):

$$MTBF = T = m + r .$$

Let

$$\text{Availability of the component} = A = m/T = m/m+r .$$

But $m = \frac{1}{\lambda}$ and $r = \frac{1}{\mu}$ so we compute

$$A = \mu/\mu + \lambda \quad [5.9]$$

Similarly;

$$\begin{aligned} \text{Unavailability of the component} &= U = r/T = r/m+r \\ &= \lambda/\mu + \lambda \end{aligned} \quad [5.10]$$

If M components are available and the remaining N-M components are not available in a system state B_j , the steady state probability of the state is calculated by the product rule of probability:

$$P(B_j) = (A_1 \cdot A_2 \dots \text{for M components}) \cdot (U_1 \cdot U_2 \dots \text{for N-M components})$$

When outdoor components such as transmission lines are included in the study, a changing environment consisting of normal and stormy weather should be considered (20). The probability of the system states in this case should be evaluated by the Markov Approach (23) since the simple product rule of probability is not valid. As an approximation the overall failure rate (normal and stormy weather) of the components can be used.

VI. COMPOSITE SYSTEM RELIABILITY EVALUATION (24, 25)

A. The Conditional Probability Approach to Composite Systems

If two events designated A and B are considered to be independent, then

$$P(A \cap B) = P(A) \cdot P(B) \quad [6.1]$$

where

$P(A \cap B)$ = Probability of A and B occurring simultaneously.

$P(A)$ = Probability of A occurring.

$P(B)$ = Probability of B occurring.

If event A is not independent and $P(A|B)$ denotes the conditional probability that A occurs given that B has occurred and

$$P(A \cap B) = P(A|B) \cdot P(B). \quad [6.2]$$

If the occurrence of A is dependent upon a number of events B_j , which are mutually exclusive (i.e., only one of the events B_j can occur at the time), we may write

$$P(A) = \sum_j P(A|B_j) \cdot P(B_j) \quad [6.3]$$

If the occurrence of A is dependent upon only two mutually exclusive events for component B, success and failure, designate B_x and B_y , respectively,

$$P(A) = P(A|B_x) \cdot P(B_x) + P(A|B_y) \cdot P(B_y) \quad [6.4]$$

With respect to reliability this can be expressed as

$$P(\text{system failure}) = P(\text{system failure if B is good}) \cdot P(B_x) \\ + P(\text{system failure if B is bad}) \cdot P(B_y)$$

B. Simple System Application (24)

Consider a simple system consisting of a generating station with two parallel transmission lines feeding a single load as shown in Figure 6.1.

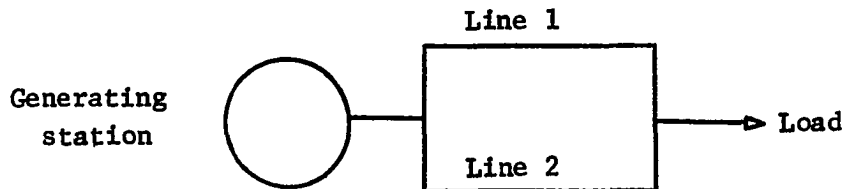


Figure 6.1. System configuration

Define:

P_G = Probability of the generating capacity outage exceeding the reserve capacity (a cumulative probability figure obtained from the capacity outage probability table).

P_L = Probability of load at a bus exceeding the maximum load that can be supplied at that bus without failure. For the simple one line radial system $P_L(1)$ can be interpreted as a probability that the load will exceed the carrying capacity of Line 1.

Q_S = Probability of system failure to serve a load at a bus.

R_{L1}, Q_{L1} = Probability of line availability and line outage, respectively, for Line 1.

R_{L2}, Q_{L2} = Probability of line availability and line outage, respectively, for Line 2.

In the simple system of Figure 6.1 the following events are possible, viz.,

1. line 1 in: (a) line 2 in
(b) line 2 out

or the mutually exclusive event

2. line 1 out: (a) line 2 in
(b) line 2 out.

Then

$$P(\text{system failure}) = P(\text{system failure/line 1 in}) \cdot P(\text{line 1 in}) \\ + P(\text{system failure/line 1 out}) \cdot P(\text{line 1 out}).$$

But by definition

$$P(\text{system failure/line 1 in}) \triangleq Q_S(L1 \text{ in}) \\ P(\text{system failure/line 1 out}) \triangleq Q_S(L1 \text{ out}) \\ P(\text{line 1 in}) \triangleq R_{L1} \\ P(\text{line 1 out}) \triangleq Q_{L1}.$$

Thus, by direct substitution

$$P(\text{system failure}) = Q_S(L1 \text{ in}) \cdot R_{L1} + Q_S(L1 \text{ out}) \cdot Q_{L1}.$$

Now the probability of system failure when line 1 is in and the probability of system failure when line 1 is out depend on whether line 2 is in or out. That is

$$\begin{aligned}
 P(\text{sys. fail./line 1 in}) &= P(\text{sys. fail. when line 1 in/line 2 in}) \\
 &\quad \cdot R_{L2} \\
 &+ P(\text{sys. fail. when line 1 in/line 2 out}) \\
 &\quad \cdot Q_{L2}
 \end{aligned}$$

A notation $Q_S(L1 \text{ in}, L2 \text{ in})$ will be used to represent the probability of system failure when line 1 in and line 2 in. Then

$$Q_S(L1 \text{ in}) = Q_S(L1 \text{ in}, L2 \text{ in}) \cdot R_{L2} + Q_S(L1 \text{ in}, L2 \text{ out}) \cdot Q_{L2} .$$

The same procedure will be repeated for $Q_S(L1 \text{ out})$.

Now the probability of system failure when a combination of events 1(a), 1(b), 2(a), and 2(b) occurs depends on the capacity deficiency probability (P_G) and the line capacity probability (P_L), e.g.

$$Q_S(L1 \text{ in}, L2 \text{ out}) = P_G + P_L(1) - P_G \cdot P_L(1) .$$

Obviously

$$Q_S(L1 \text{ out}, L2 \text{ out}) = 1.0$$

The expression of the probability of system failure Q_S in terms of R_{L1} , R_{L2} , Q_{L1} , Q_{L2} , P_G , $P_L(1)$, $P_L(2)$, and $P_L(1, 2)$ will be derived below.

Starting with

$$Q_S = Q_S(L1 \text{ in}) \cdot R_{L1} + Q_S(L1 \text{ out}) \cdot Q_{L1} \quad [6.5]$$

For L1 in

$$Q_S(L1 \text{ in}) = Q_S(L1 \text{ in}, L2 \text{ in}) \cdot R_{L2} + Q_S(L1 \text{ in}, L2 \text{ out}) \cdot Q_{L2} \quad [6.6]$$

For L1 in and L2 in

$$Q_S(L1 \text{ in}, L2 \text{ in}) = P_G + P_L(1, 2) - P_G \cdot P_L(1, 2) \quad [6.7]$$

The probabilities of capacity deficiencies P_G and transmission inadequacies are independent.

For L1 in and L2 out

$$Q_S(L1 \text{ in}, L2 \text{ out}) = P_G + P_L(1) - P_G \cdot P_L(1) \quad [6.8]$$

Therefore, by substituting Equations 6.7 and 6.8 in Equation 6.6, for L1 in:

$$\begin{aligned} Q_S(L1 \text{ in}) &= [P_G + P_L(1, 2) - P_G \cdot P_L(1, 2)] \cdot R_{L2} \\ &\quad + [P_G + P_L(1) - P_G \cdot P_L(1)] \cdot Q_{L2} \end{aligned} \quad [6.9]$$

For L1 out

$$Q_S(L1 \text{ out}) = Q_S(L1 \text{ out}, L2 \text{ in}) \cdot R_{L2} + Q_S(L1 \text{ out}, L2 \text{ out}) \cdot Q_{L2} \quad [6.10]$$

$$= [P_G + P_L(2) - P_G \cdot P_L(2)] \cdot R_{L2} + [1]Q_{L2} \quad [6.11]$$

By substituting Equations 6.9 and 6.11 back in Equation 6.5, we obtain for the complete system:

$$\begin{aligned}
Q_S &= [\{P_G + P_L(1, 2) - P_G \cdot P_L(1, 2)\} \cdot R_{L2} \\
&\quad + \{P_G + P_L(1) - P_G \cdot P_L(1)\} \cdot Q_{L2}] \cdot R_{L1} \\
&\quad + [\{P_G + P_L(2) - P_G \cdot P_L(2)\} \cdot R_{L2} + Q_{L2}] \cdot Q_{L1} \\
&= R_{L1} \cdot R_{L2} \cdot \{P_G + P_L(1, 2) - P_G \cdot P_L(1, 2)\} \\
&\quad + R_{L1} \cdot Q_{L2} \cdot \{P_G + P_L(1) - P_G \cdot P_L(1)\} \\
&\quad + R_{L2} \cdot Q_{L1} \cdot \{P_G + P_L(2) - P_G \cdot P_L(2)\} \\
&\quad + Q_{L2} \cdot Q_{L1} \cdot (1) \tag{6.12}
\end{aligned}$$

The first term in each line of Equation 6.12 can be interpreted as follows:

$R_{L1} \cdot R_{L2}$ = Probability of both lines being available.

$R_{L1} \cdot Q_{L2}$ = Probability of Line 1 being available and
Line 2 being unavailable.

$Q_{L1} \cdot R_{L2}$ = Probability of Line 1 being unavailable and
Line 2 being available.

$Q_{L1} \cdot Q_{L2}$ = Probability of both lines being unavailable.

These quantities represent the steady state probabilities of existence of the outage states due to outage conditions in the transmission network. Also, the quantities in the brackets in Equation 6.12 represent the conditional probabilities of system failure at the load for that particular outage state. Note that

the conditional probability of system failure with both lines out is unity.

Define

- B_j = Outage state in the network.
 $P(B_j)$ = Probability of existence of state B_j .
 $P_G(B_j)$ = Probability of the generating capacity exceeding the reserve capacity when outage state B_j exists.
 $P_{Lk}(B_j)$ = Probability that the load at bus k exceeds the maximum load that can be supplied to that bus under outage state B_j .

Then the probability of system failure at bus k is

$$Q_{Sk} = \sum_j P(B_j) [P_G(B_j) + P_{Lk}(B_j) - P_G(B_j)P_{Lk}(B_j)] \quad [6.13]$$

If the generating units are treated as 100% available [$P_G(B_j) = 0$], the probability of system failure at bus k becomes

$$Q_{Sk} = \sum_j [P(B_j) \cdot P_{Lk}(B_j)] \quad [6.14]$$

C. Service Quality Criterion for Reliability (24)

Continuity is not an acceptable single reliability criterion. The definition of a breach of continuity can be extended to include a breach of quality of service. During a line outage a low voltage condition may exist at a load point. This is not actually a breach of continuity though the voltage level may be considerably lower than a desired minimum voltage. If bounds are placed upon desirable voltage

levels at each point in the system, any departure from these ranges can be classified as a breach of service, which now includes breach of continuity or quality. This would not include voltage transients caused by system disturbances unless the voltage persisted for a defined period of time in the unacceptable region.

In a composite system including both generation and transmission facilities, there are a number of possible outage combinations of lines, transformers and generating units. Each outage condition has a probability of existence. Under each outage condition there is a maximum load at each bus that can be supplied without violating the service quality criterion. The unacceptable service qualities might include low voltages, equipment overloads, and generating capacity inadequacy. The probability that the load will exceed these maxima can be determined from the load probability distribution for the bus in question. This is a conditional probability for the maximum load given that a certain outage condition has occurred. The maximum load that can be supplied at a bus can be obtained for any given outage condition in the transmission system. The system generating facilities can be included by developing a capacity outage probability table for all the units within the system. Then the probability of the load at the bus exceeding this maximum value is combined with the probability that the available system generation capacity is insufficient to meet the total system load. Generation and transmission outage conditions are considered as two independent events resulting in failure at a bus.

D. Assumption in Reliability Calculations

Some of the assumptions which were made in deriving the reliability calculation method have been mentioned in preceding sections. These assumptions together with other pertinent assumptions are listed as follows for ease of reference.

1. The system contains components which are operating within their useful life period (i.e., the failure rate λ is constant).
2. Neglect the effect of fluctuating environment. This is not a good assumption because during stormy periods, environmental conditions may be so severe as to result in higher component failure rates than those prevailing during nonstorm periods. But, as a matter of illustrating the method, this assumption is made.
3. Times of failure (Periods between failures) and repair times are exponentially distributed and independent of each other; that is,

$$\text{probability [time to failure} > t] = e^{-\lambda t}.$$
4. Each component can exist only in two states, either available or unavailable (under forced outage) and that the component states are independent of each other.
5. The method assumes system failure at a bus if
 - a) the bus is isolated and there is no generation at the bus
 - b) the bus voltage exceeds the minimum (0.95 p.u.) or maximum (1.1 p.u.) acceptable values.

6. Each bus load can be described by a probability distribution, obtained by analysis of the historical load data. In this thesis a normalized load duration curve of the bus will be used to represent the probability distribution. A normalized load duration curve is a relation between bus load and duration (time) that load exceeds the indicated value. Bus load from the curve will be in MW and duration in p.u. of 24 hours. The normalized load duration will be approximated by a single straight line (see Figure 6.3).

7. For a given system load level, all the bus loads are at the same duration at that of the system load. For example, system load L_S , shown in Figure 6.3, is distributed to the individual load buses as L_3 , L_5 , and L_6 and the corresponding duration for each bus as well as the system is P_L .

In load flow studies to determine the maximum load that can be supplied at a bus, the voltage level is checked against the acceptable limits at each load level. Different buses experience difficulty under certain conditions in terms of low voltages. The maximum load, $M_3(B_j)$, is the load that can be supplied at bus 3 with the system in state B_j before bus 3 experiences difficulty. The duration $P_{L3}(B_j)$, in Figure 6.3, which is the conditional probability of load at bus 3 exceeding the maximum value $M_3(B_j)$ can be obtained as outlined below.

E. Reliability Study of a Simple System

The application of Equation 6.13 is illustrated by evaluating the probability of system failure at buses 3, 5, and 6 of the system

configuration shown in Figure 6.2. This is the same system described in Chapter 4. Assume the whole system (2 areas) is operating at peak load. The forced outage parameters of the transmission lines and the transformers are shown in Table F.1 (Appendix F). The generating units are treated as 100% available because the reliability due to transmission component failures is the primary interest in this study. Then $P_G(1)$ and $P_G(2)$ are both zero. The probabilities of existence of outage states due to outage conditions in the transmission network, $P(B_j)$, are calculated by the product rule of probability neglecting stormy weather (assumption #2). The results would be similar to the first term in each line of Equation 6.12, but the number of probability terms in those individual products depends on the total number of transmission branches (paths) in the system. The transmission paths include both transmission lines and transformers. The probability terms (availability or unavailability) made up those individual products depend on the outage conditions. The outage conditions assumed in this thesis include both single outages (one-component outages) and double outages (two simultaneous component outages). All other multiple outages other than double outage are assumed negligible. This is a good assumption as the probability of triple or higher order contingencies is usually very small.

For example, the probability of existence of the outage state due to an outage on branch (path) 5, which is a transformer outage, $P(B_j)$, is calculated below.

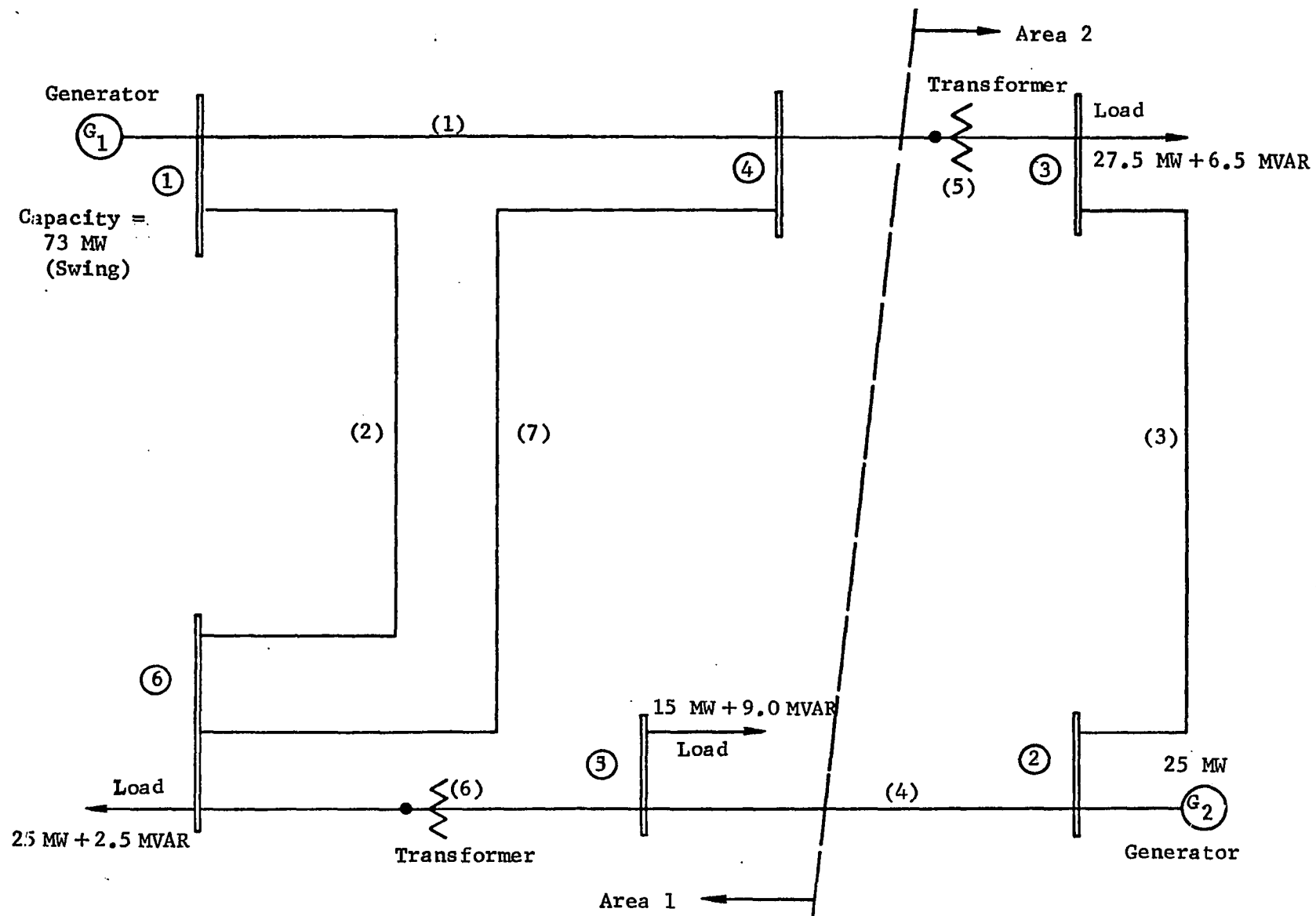


Figure 6.2. A simple system configuration

$$P(B_j) = R_{L1} \cdot R_{L2} \cdot R_{L3} \cdot R_{L4} \cdot Q_{L5} \cdot R_{L6} \quad [6.15]$$

where

Q_{Lk} = The unavailability of a line or transformer on the path k.

R_{Lk} = The reliability of path k.

The calculation of the probabilities of existence of outage states is shown in Table F.2 for the maximum number of 7 paths including all the transmission lines and the transformers. To study the effect on the reliability of the system due to the line connecting bus 4 and bus 6, the line (path 7) is discarded from the system and the probabilities of various system states are recalculated in Table F.3. The normalized load duration curves at bus 3, 5, and 6 are arbitrarily assumed as shown in Figure 6.3. There is no technical basis for this assumption. The only numerical basis is that the load levels at each load bus should correspond to the levels shown in Figures C.1 and C.2. The load levels for different durations are shown in Table F.4.

The determination of the maximum load supplied to those load buses for different system states requires load flow analyzes with certain additional assumptions.

1. The normalized load duration curve is divided into six steps. Under each outage state B_j , if the voltage at a load bus k exceeds the minimum (0.95 p.u.) or maximum (1.1 p.u.) acceptable values at any of the load levels, $P_{Lk}(B_j)$ is taken as the average of the duration of the load level at which the load bus failed and the previous load level.

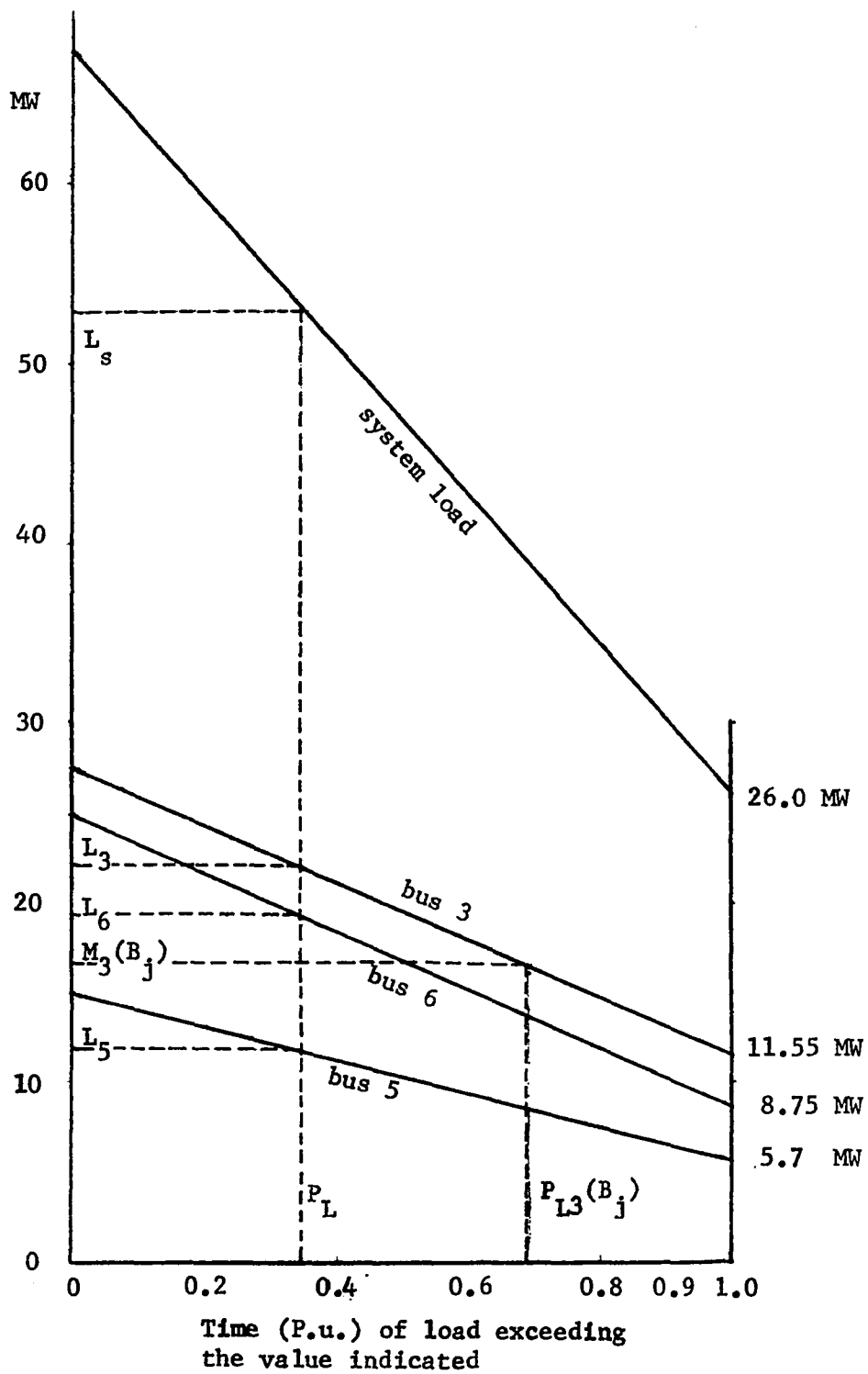


Figure 6.3. Normalized load duration curves

Assumed for the system shown in Figure 6.1

This is an approximate way to facilitate this calculation. The more steps the curve is divided into the more accurate the results.

2. If load bus k is isolated from the network due to some outage condition, $P_{Lk}(B_j)$ is equal to 1.0 for that bus.

3. If the original swing bus is isolated from the network due to some outage condition, another bus is selected as the swing bus.

4. Assume no line and transformers are overloaded at any load level.

The detail of reliability calculations is shown in Table F.5 for the system with 7 paths and in Table F.6 for the system with 6 paths (path 7 neglected). The summary of the results is shown in Table 6.1.

Table 6.1. Summary of results

Number of paths	Probability of system failure		
	Bus 6	Bus 3	Bus 5
7	.189808386	.280473264	.281247628
6	.192152045	.282398177	.283880021

In the 6 path case, all the loads have a similar physical arrangement. A line and a transformer are terminated at each load bus. The probability of system failure at bus 6 is less than at the other buses because bus 6 is directly connected to the swing bus. Because of the system configuration and the location of the chosen swing machine, Transformer 5 has more effect on the reliability of the system than Transformer 6, although they have the same outage parameters. It is

apparent that with such a small system and with the assumption that there is no component overloaded, the location of the swing machine plays an important part in the reliability of the system.

When path 7 (the line connecting bus 4 and bus 6) is added to the system, the availability of the path will be included as an additional factor in the product [similar to Equation 6.15] to determine the probability of existence of an outage state which does not involve that path (path 7 in this case). Obviously, the probability will be decreased because the availability of path 7 is less than 1.0. It is also obvious that the maximum load that can be supplied to each load bus in the system with 7 paths should be equal to or greater than the maximum load in the system with 6 paths; or in other words, the conditional probabilities at the load buses should be less than or equal to the corresponding probabilities in the system with 7 paths than in the system with 6 paths. For these reasons the probabilities of system failure at the load buses will be the same or decreased with the path 7 in service. The probabilities will never be greater with the path 7 in service.

The result shown in Table 6.1 provides a good indication of the effect on the probabilities of system failure of having the path 7 in the system. With the reliability benefit of a transmission line defined in Chapter 3, it is obvious that the load buses obtain the reliability benefit from the line connecting bus 4 and bus 6. This result will be used to allocate a portion of fixed charges of the line due to reliability considerations.

The calculation could have been done more easily and in less time with a computer program developed for this particular purpose (27). After getting all the results from load flow analysis, most of the calculation was done by an electronic desk calculator because the mathematics is not laborious as far as the sample system is concerned. But it would be a very time-consuming job if the system were large.

The number of load flow runs required depends on the number of paths in the system. Twenty-nine load flow runs are required for a system with 7 paths and 22 runs for a system with 6 paths. This is for a single load level and with only single and double outages considered.

If the voltage criterion is the only criterion to indicate bus failures, the location of the swing machine will contribute greatly to the reliability. A load bus which is directly connected to the swing machine will appear to be more reliable. At a load bus which is electrically remote from the swing machine, the load will have less chance of being supplied adequately, because of voltage-drop problem, if the generator which is usually supplying the load is isolated by a transmission outage.

For more comprehensive study the equipment overload criterion and the generation deficiency criterion should be considered also.

If the equipment overload criterion is included in the study, then the load bus will have greater reliability if a large number of paths are connected to a load bus. This is because power can flow to the load over these alternate paths without overloading them.

If the generation deficiency criterion is included, the system capability to supply its load will be less if there are a small number of very large generators, than if there are a larger number of smaller generators in the system. This is because a larger generator has less availability than a smaller generator and when a large generator is on outage the system will lose a large percentage of generation in the system.

If the study is to be made for the same purpose for a real system of, say, 100 nodes, the computation need not be made exhaustively for all lines in the system. Practically the lines of primary interest are those high voltage lines which are usually heavy loaded and important as far as the system operation is concerned. The study does not have to be made with all lines in the system. The system can be reduced in size to include only high-voltage lines which are primarily the backbone of the system.

VII. ALLOCATION OF FIXED CHARGES

There are transmission problems in setting up and expanding a power pool. Transmission helps everybody, more or less. Generally, each member of the power pool builds its own transmission lines to serve customers in its service territory. Occasionally, a jointly used transmission line, needed for the benefits of the power pool, is planned and the difficulty of agreeing on ownership may arise.

Fixed charges of transmission lines include return, taxes, depreciation, insurance, and maintenance cost. The fixed charges are allocated to members of the pool in proportion to ownership. Several possible bases for ownership of lines are as follows:

1. The ownership may be divided equally among the members.
2. The ownership may be proportional to the installed capacity requirements of each company when operating separately.
3. The ownership may be proportional to the peak load of each for separate operation.
4. The ownership may be proportional to a combination of energy consumed, average load, and the difference between the maximum load and the average load of each company (excess demand).
5. The ownership may be proportional to the distribution and the reliability benefits each member company derives from using the transmission system.

The first base is equitable in a very rare circumstance, i.e., when these companies have nearly the same size and similar essential

characteristics. The bases 2 and 3 for ownership are not equitable as far as a transmission line is concerned because the line is not planned for construction on these bases. If the problem was one of adding generation units, these bases may be equitable.

The base 4 seems more equitable if one can find an appropriate way to formalize the idea. The phantom customer method of allocating fixed charges is suggested. This method allocates the costs from both the energy and the power point of views. This method may be appropriate for operating purposes on a day-by-day or month-by-month basis. It is easy to compute if we have all the data.

For planning purposes, however, base 5 seems to be more logical and equitable.

A. Fixed Cost Allocation Based on Energy and Excess Demand Considerations

The original concept of the phantom method is described in detail in Appendix A. The idea of cost allocation based on energy and excess demand considerations is applied to the 6 bus system (Appendix G). The system is supposed to be a power pool with two members (Area 1 and Area 2). The fixed charges of the line connecting bus 4 and bus 6, path 7, are allocated between these members on the basis of energy and excess demand each member consumes.

The fixed charges of the transmission line in equivalent MW are divided into two parts. A part of the fixed charges of the line contributes to energy, which is called an energy charge of the line.

This energy charge in MW is equivalent to the power flow in that particular line when the pool is operating at the average load level. Another part of the fixed charges contributes to excess demand which is called a phantom demand charge of the line. The demand charge in MW is equivalent to the difference between the power flows in the line at the pool peak load level and at the pool average load level. The difference should have only a positive value in order to be significant. A negative value means that there is no excess demand at the time of pool peak.

The equivalent energy charge in MW is allocated to Area 1 and 2 in proportion to their individual real power consumed at the average pool load level. The equivalent phantom demand charge in MW is allocated to the members in proportion to the increment of flow in that particular line due to the load conditions (load conditions A and B in Table G.2). Again the increment should be positive in order to be significant. A negative value of the increment indicates that the member company which experiences the load change causes less flow in the line at the pool peak level than the flow at the pool average load level. Then the total charge in MW to each member is the sum of contributions in MW from that member company to the energy charge and to the phantom demand charge.

Define

P_{a7} = Magnitude of real power flow in path 7 at a pool average load level.

P_{p7} = Magnitude of real power flow in path 7 at a pool peak load level.

L_1 = Total load in MW of Area 1 at the pool average load level.

L_2 = Total load in MW of Area 2 at the pool average load level.

ΔP_{C7} = Increment of flow in path 7 due to any load condition designated as "C" (Positive value only).

F_7 = Fixed charges of path 7 in MW.

F_{17} = Fixed charges of path 7 allocated to Area 1.

F_{27} = Fixed charges of path 7 allocated to Area 2.

F_{e7} = The part of F_7 due to energy charge in MW.

F_{d7} = The part of F_7 due to phantom demand charge in MW.

Then

$$F_7 = F_{e7} + F_{d7} . \quad [7.1]$$

But

$$F_{e7} \triangleq P_{a7} \quad [7.2]$$

and

$$F_{d7} \triangleq P_{p7} - P_{a7} . \quad [7.3]$$

It is proposed that

$$F_{17} = \left(\frac{L_1}{L_1 + L_2} \right) P_{a7} + \frac{\Delta P_{B7}}{\Delta P_{A7} + \Delta P_{B7}} (P_{p7} - P_{a7}) \quad [7.4]$$

and

$$F_{27} = \left(\frac{L_2}{L_1 + L_2} \right) P_{a7} + \frac{\Delta P_{A7}}{\Delta P_{A7} + \Delta P_{B7}} (P_{p7} - P_{a7}) . \quad [7.5]$$

By substituting the values of the variables in Equations 7.4 and 7.5, we have

$$F_{17} = \left(\frac{26}{26 + 17.9} \right) \times 2.78 + \frac{5.45}{0.0 + 5.45} \times (2.08)$$

$$= 1.35 + 2.08 = 3.43 \text{ MW}$$

$$F_{27} = \left(\frac{17.4}{26 + 17.9} \right) \times 2.78 + \frac{0.0}{5.45} (2.08)$$

$$= 0.93 \text{ MW}$$

That is,

$$F_{17} = 79\% \text{ of } F_7$$

and

$$F_{27} = 21\% \text{ of } F_7 .$$

In a general case with n member companies in a power pool the general equation of fixed charges of path x allocated to Area i is

$$F_{ix} = \left(\frac{L_i}{\sum_1^n L_i} \right) P_{ax} + \left(\frac{\Delta P_{cx}}{\sum \Delta P_{cx}} \right) (P_{px} - P_{ax}) . \quad [7.6]$$

B. Fixed Cost Allocation Based on Distribution and Reliability Benefits

It is usually accepted that transmission lines help in distributing power to the load and contribute to the reliability of service. The reliance on the transmission in a modern pool is more crucial than just providing help in an emergency. It applies whenever generating capacity is short for any reason, for example during the refueling of a nuclear

unit, and physical back up has now become as necessary to reliable operation as emergency assistance. This is the reason why the reliability benefit should be involved in the allocation. For these reasons method 5 seems to be more equitable than the other four methods for planning purposes. Thus, method 5 will be applied to allocate fixed charges of jointly used transmission lines using a technique that could be used by the planning engineer.

Consider the simple 6 bus system used to realize the distribution benefit (Appendix C) and the reliability benefit (Appendix F). It will be assumed that the line connecting bus 4 and bus 6 in Area 1 is to be a jointly used transmission line. The reason that it is in the "territory" of Area 1 makes Area 2 reluctant to own any part of it. However, it will be shown that Area 2 gains both distribution and reliability benefits from this line and should pay a portion of the fixed charges. The dc model method will be used throughout the following computations.

From Table C.7, when the load in Area 2 is increased,

$$\Delta P_{4-6} = - 2.76 \text{ MW} .$$

From Table C.9, when the loads in Area 1 are increased,

$$\Delta P_{4-6} = 4.64 \text{ MW} .$$

When the load in Area 2 is increased, the power flow in the line, which is normally from bus 4 to bus 6, is decreased. So Area 2 does not obtain any distribution benefit from the line at all (by definition of distribution benefit defined in Chapter 3) although it is entitled up to $\Delta P_{4-6} = 0.0$ without paying for the distribution benefit. On the

other hand, when the loads in Area 1 are increased, the power flow in the line is increased. So Area 1 gets all the distribution benefit from this line.

To evaluate the reliability benefit, consider the results of Table 6.1 which are rewritten in Table 7.1 with the contributions for individual loads indicated.

Table 7.1. Reliability benefit

Bus no.	Area	<u>Prob. of system failure</u>		<u>Area prob.</u>		Difference
		6 paths	7 paths	6 paths	7 paths	
5	1	.283880021	.281247628			
6	1	.192152045	.189803386	.476032066	.471056014	.004976052
3	2	.282398177	.280473264	.282398177	.280473264	.001924913

The difference of the area probabilities of system failure with and without path 7 is shown in the last column. This difference indicates the reliability benefit that each area obtains from path 7. As expected, Area 1 obtains more reliability benefits from path 7 than Area 2 but Area 2 is also more reliable with path 7 in service. They both should make contributions for path 7 on the basis of reliability benefits.

Define

R_{17} = Reliability benefit to Area 1 from path 7.

R_{27} = Reliability benefit to Area 2 from path 7.

D_{17} = Distribution benefit to Area 1 from path 7 (MW).

D_{27} = Distribution benefit to Area 2 from path 7 (MW).

F_7 = Fixed charges of path 7 (\$).

F_{17} = Fixed charges of path 7 allocated to Area 1 (\$).

F_{27} = Fixed charges of path 7 allocated to Area 2 (\$).

F_{R7} = A part of F_7 that contributes to reliability.

F_{D7} = A part of F_7 that contributes to distribution.

Let

$$F_{R7} = C_R F_7 \quad [7.7]$$

$$F_{D7} = C_D F_7 \quad [7.8]$$

where C_R and C_D are proportionality factors.

Lacking any compelling reason to do otherwise we assume a fifty-fifty split of F_7 , that is

$$C_R = C_D = \frac{1}{2} \quad [7.9]$$

It is proposed that

$$\begin{aligned} F_{17} &= \frac{R_{17}}{R_{17} + R_{27}} \cdot F_{R7} + \frac{D_{17}}{D_{17} + D_{27}} \cdot F_{D7} \\ &= \frac{1}{2} \left(\frac{R_{17}}{R_{17} + R_{27}} + \frac{D_{17}}{D_{17} + D_{27}} \right) \cdot F_7 \end{aligned} \quad [7.10]$$

$$F_{27} = \frac{1}{2} \left(\frac{R_{27}}{R_{17} + R_{27}} + \frac{D_{27}}{D_{17} + D_{27}} \right) \cdot F_7 \quad [7.11]$$

In general case the fixed charges of path x allocated to Area i in a power pool with n member companies is

$$F_{ix} = \frac{1}{2} \left(\frac{R_{ix}}{\sum_i R_{ix}} + \frac{D_{ix}}{\sum_i D_{ix}} \right) \cdot F_x \quad [7.12]$$

By substituting the values of R_{17} , R_{27} , D_{17} , and D_{27} in Equations 7.10 and 7.11, we have

$$\begin{aligned} F_{17} &= \frac{1}{2} \left(\frac{.004976052}{.006900965} + \frac{4.64}{4.64 + 0.0} \right) \cdot F_7 \\ &= \frac{1}{2} (0.720 + 1.0) \cdot F_7 \\ &= 0.860 F_7 \end{aligned} \quad [7.13]$$

$$\begin{aligned} F_{27} &= \frac{1}{2} \left(\frac{.001924913}{.006900965} + \frac{0.0}{4.64} \right) \cdot F_7 \\ &= 0.140 F_7 \end{aligned} \quad [7.14]$$

Thus Area 1 should pay $0.860 F_7$ for path 7 and Area 2 should pay $0.140 F_7$, although path 7 is not in its own service territory. In this example Area 1 received only a reliability benefit. In other cases, and in general, both a reliability and a distribution benefit would be possible.

Compared with the results shown in Table G.4, obtained from the phantom method of fixed-charge allocation, Area 1 should pay $.79 F_7$ and Area 2 should pay $.21 F_7$.

As noted in Chapter 4, the base value to determine p.u. quantity of the distribution benefit is the total sum of the distribution benefits of that particular line to each separate area in the pool. The advantage of this base value will be seen in Equations 7.10 and 7.11. The p.u. value of $(D_{17} + D_{27})$ is 1.0 and it facilitates the calculation. Then

$$F_{17} = \frac{1}{2} \left(\frac{R_{17}}{R_{17} + R_{27}} + D_{17}(\text{p.u.}) \right) F_7 \quad [7.15]$$

and

$$F_{27} = \frac{1}{2} \left(\frac{R_{27}}{R_{17} + R_{27}} + D_{27}(\text{p.u.}) \right) F_7 . \quad [7.16]$$

It is also apparent that $(R_{17} + R_{27})$ constitutes the total reliability benefit to both areas and is in fact a base reliability benefit for the entire system. In general we let the base reliability benefit for line x be defined as

$$(\text{Base Reliability Benefit}) = \sum_i R_{ix} \quad [7.17]$$

Then the fixed charge allocation of company i for line x is

$$F_{ix} = \frac{1}{2} (R_{ix} + D_{ix}) F_x \quad [7.18]$$

where both R_{ix} and D_{ix} are in per unit.

VIII. DISCUSSION AND CONCLUSIONS

It is believed that the foregoing methods permit a comprehensive evaluation of the benefits of a transmission line for distributing power to the load and for contributing to the reliability of an area. The method recognizes many important parameters that must be considered, and computer programs can be developed to handle all calculations.

The wheeling benefit has been ignored in the method because this benefit could be realized only when the movement of power under contract is known. Such a contract depends on situations which arise between the companies and is difficult to forecast in long range planning.

Nothing has been said in this thesis about the handling of transmission losses, which can get quite involved, in a large pool. The working out of this problem is an operating matter and can be solved only by agreements to some short-cut approach.

Another method which applies the idea of the phantom customer method to allocate the fixed charges of a transmission line in a power pool has also been developed. The phantom method seems to be more equitable than other rate methods and is easy to compute. This is suggested for day-by-day operation.

It is the purpose of this research to provide a simple and equitable way to allocate fixed charges of jointly-used transmission lines based on benefits obtained. It is believed that a new method has been developed in this thesis, whereby power pool members can share transmission responsibility equitably on the basis of the benefits they

derive from the transmission system. The method is believed to be fair to all parties and may be applied without guess work or arbitrary decisions as to the alleged merits of a given line. Furthermore, the method yields a numerical result and a quantitative measure of the two important pool benefits, distribution, and reliability.

The method is suggested for long range system planning. The method should be applied early in transmission planning as possible, where it will have the least impact, so that it will not be a sudden shock to the management as far as the expense is concerned. Such an outcome would not mean an extra expense to a company but it would mean that they admit that they should contribute to those facilities from which they derive known benefits.

To apply the method to a real power pool, a few comments should be made. First, the size of the matrix to be inverted may be too large to be handled properly by ordinary numerical techniques. This problem can be taken care of by reducing the size of the pool to a smaller one and keeping only the high-voltage lines in the pool for the study. This could be done because the line of primary interest should be an important line operating at a high voltage level and heavily loaded.

Secondly, the method suggested to determine the distribution benefits depends on a predicted peak load of the pool. It may be difficult to forecast the load level of each area in the pool at the time of the forecasted pool peak.

Thirdly, another problem which may be difficult is to forecast the load level of each system in the pool at the time that the pool is

operating at the average load level. The average load level of the pool may be a single acceptable quantity but there are many combinations of the loads in each area which make up the quantity. Everybody may try to choose the combination which costs him the least.

IX. SUGGESTIONS FOR FUTURE WORK

The purpose of this research was to investigate allocation techniques and apply them to a small sample system. There are some points that were not considered in this thesis because they might unnecessarily complicate the problem and obscure the real purpose of this research.

One area for further study is the problem of including the effect of generation deficiency into the reliability benefit. As the generation is not 100 percent available throughout the whole system it should be of interest to include both transmission outages and generation outages in the outage states of the system.

Also, the effect of the size of the increment of load on the distribution benefit might be of interest. This effect could be studied either by a mathematical analysis of the sensitivity matrix or by varying the size of the increment in order to find its relation to the distribution benefit. This latter approach would require a detailed computer analysis.

Finally there is much work to be done in making the methods suggested in this thesis available as proven operating computer programs.

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XII. APPENDIX A: ALLOCATION OF DEMAND COSTS

A. Methods of Allocating Demand Costs

1. The energy method

The simplest and one of the most commonly used methods allocates the demand costs in proportion to the energy used by each class of consumer during a former period, such as a typical month or year.

Such a method is simple because the values of energy used by the various classes during past periods are generally available from records. If all customers had 100% load factor, the method would be perfectly fair to all customers.

This method is inequitable because demand costs are not basically proportional to the energy used, but rather to the maximum demand of the class of consumers. The class with the large use of energy would be over-burdened.

2. The peak responsibility method

This method allocates the demand costs in proportion to the demand made by each class of consumer on the system at the time of the system maximum demand. The method is an attempt to place the burden upon those classes of consumers responsible for the large amount of investment required to serve the peak load period. If a company serves classes of consumers whose peaks are coincident in forming the annual peak on the company's system, the peak responsibility method is fair and just. In the early days, when the principal load was lighting, this condition existed.

It is obviously unfair to charge one class of customers who happen to use energy at the time of the annual system peak with all of the demand costs and let the other customers use the equipment for nothing. This is not only unfair but it is impracticable, for a peak due to one class of customers may coincide with the system annual peak and in the following year the system peak may be caused by different classes.

3. The maximum demand method

The criticism of the peak responsibility method suggested that the demand costs may be more equitably allocated by the ratio of the maximum demand of the class under consideration to the summation of the maximum demands of all classes.

This method gives correct results only in certain isolated cases. If the customer's peaks coincide, this method agrees with the peak responsibility. In cases where the customer maximum demands are not coincident, there is no overlapping of curves and the load factors (average load/maximum load) of all customers are the same, this method is applicable and the results are just and fair.

But there are two important aspects that are neglected in the method. First, it neglects the important item of time that the peaks occur. Second, it entirely neglects the energy required by those classes.

This method will encourage long hour use of the individual demand because all consumers who have a load factor higher than average will be charged too little, and all who have a load factor lower than the

average will be charged too much. But it is not justifiable to serve long hour users below cost.

4. The Greene's method

So impressive are the errors in the maximum demand method that a new method was developed. This method uses a combination of the maximum demand and the energy methods. Part of the demand costs are a direct function of the maximum demands and the remainder is a direct function of energy. The proper values can be obtained by solving two simple equations in which:

x = The cost per kilowatt-hour of that portion of the demand costs that functions with the kilowatt-hours supplied the consumers.

y = The demand cost per kilowatt of that portion of the demand costs that function with the maximum demand of the consumers.

D = The sum of the consumers maximum demands.

P = The maximum coincident demand or peak responsibility of all consumers on the sources of supply.

K = The kilowatt-hours used by all the consumers in a year.

C = The total annual demand costs of all the consumers.

8670 = The kilowatt-hours in a year for a 1-kW load operated at 100% power factor and 100% load factor (number of hours in a year).

The equations are

$$Kx + Dy = C$$

$$8760x + y = C/P$$

Without any doubt it is a fairer method than any of those previously discussed. This method is simple to calculate. But it neglects one very important item, and this is the time at which the individual maximum demands occur, although it does recognize the duration of such load.

Also, if all customers fail to have the same percentage kilowatt-hour loss between the generating plant and the consumers' meters, the kilowatt-hours generated, not the kilowatt-hours consumed, should be used.

5. The Eisenmenger theory

Eisenmenger made a most elaborate study of central station load curves and their relative contribution to the demand costs of the system. He advocated the following simplified method of allocation. Eisenmenger's method will be found more equitable than the first three previous methods, because it takes into consideration not only the so-called on-peak but also the off-peak load of the various consumer classes and their duration. If we let the proportionality factors of the classes of consumers sharing the annual demand costs be represented by F_{class} and the total demand costs be divided by their sum, then the demand costs to be allocated to each class will be $\frac{F_{\text{class}}}{\sum F_{\text{class}}}$.

(total demand costs) . It remains, therefore, only to find equitable

values of F_{class} .

From an elaborate graphical analysis of many load curves, the following empirical formula has been developed for determining these factors:

$$\begin{aligned}
 F_{\text{class}} = & MD_{\text{class}} \times \frac{\% SP_{\text{class}}}{100} \\
 & + MD_{\text{class}} \times \left(1.0 - \frac{\% SP_{\text{class}}}{100}\right) \times \frac{\text{Peak hours}}{24} \\
 & + (MD_{\text{class}} \text{ OP}) \times \frac{\text{OP hours}}{24} .
 \end{aligned}$$

This states that the proportionality factor of a class is equal to the sum of the following constants:

1. Maximum demand of class (MD_{class}) times percentage of station peak of class, ($\% SP_{\text{class}}/100$):
2. Maximum demand of class times remainder percentage of station peak of class times ratio of hours per day to 24 hours during which the class peak and station peak coincide, plus;
3. Maximum demand off-peak ($MD_{\text{class}} \text{ OP}$) of class times ratio of hours per day to 24 hours during which the class peak and the station peak do not overlap.

For off-peak consumers the Eisenmenger theory gives correct results, but it does not divide the demand costs correctly among those consumers who are on at the time of the station peak. It is Eisenmenger's contention that every customer who is on at the time of station peak

contributes to that peak. However, the favorable 100% load factor consumer has no peak in his individual demand curve. Therefore, the method burdens rather heavily this favorable class of consumer who has a steady load.

6. The phantom method

If a public utility could operate steadily at its maximum demand for 24 hours a day everyday, i.e., at 100% load factor, its investment in equipment would be used most economically. The loss of any customer will affect the load factor, or efficiency of plant use, regardless of the fact that one might have twice the demand of the other. This was the conclusion of Hills when he asserted that a fair and just division of cost will be on a kilowatt-hour basis, for every block of energy used is just as important as every other block of the same size as far as costs to the central station is concerned. So, with a plant operating at 100% load factor, the demand costs divided by the number of kilowatt-hours generated and multiplied by the consumption of each customer at the generating plant will give the true demand costs that should be allocated to each customer. The demand costs per kilowatt-hour under these conditions may be easily calculated as

$$\frac{\text{Total demand costs per annum}}{(\text{Max demand}_{\text{station}}) (24) (365)}$$

In actual practice, the load factor is usually not 100%. But the line of reasoning will still apply if account is taken of an imaginary customer needed to give the ideal condition. The demand costs are

divided among the groups of customers according to their kilowatt-hour consumption, charging this phantom customer in the same way as the real customers. So now the problem is to divide the bill of this phantom customer, which would be required to operate the existing plant at 100% load factor, among the existing customers in an equitable manner.

Certainly the customers who already have a 100% load factor are not responsible for the bill and neither are those customers who are off-peak, for they are doing their share toward reducing the size of this phantom. Those customers that cause the peak are responsible because they use more than their average demand during the period of that peak load and their degree of responsibility is limited to the excess demand during the period of the station peak load over the average demand.

In many cases it may be that there is not only one station peak during the year, due to one set of conditions, but perhaps two or more peaks at other times due to different groups of customers or under different conditions. It often happens that the annual station peak is just as likely to occur due to one group of customers as another. This is a case where the phantom method can be applied with accuracy and ease.

7. The weighted peak method

This method allocates the demand costs to the various classes of consumer according to the share of each class in the total weighted peak. The weighted peak of any class of consumer is taken as equal to

the demand of that class at the time of the plant peak plus a fraction of the difference between the maximum demand of that class of consumers and its demand at the time of the plant peak. This fraction that is added is the ratio of the plant demand at the time of the class maximum demand as compared with the total peak demand.

B. Numerical Example

The 6 methods above will be analyzed and compared by way of a numerical example below. Suppose a typical load duration curve of a system is shown in Figure A.1. There are five groups of customers, A, B, C, D, and E. The load description of each group of customers is shown in Table A.1.

Table A.1. Load description

	KW-Hr.	KW demand on peak	KW Max. demand	Hours on station peak	Total hours
A	600,000	10,000	10,000	2	2
B	1,800,000	8,000	8,000	2	24
C	1,440,000	2,000	2,000	2	24
D	3,000,000	0	5,000	0	20
E	600,000	0	10,000	0	2
Total	7,440,000	20,000	35,000		

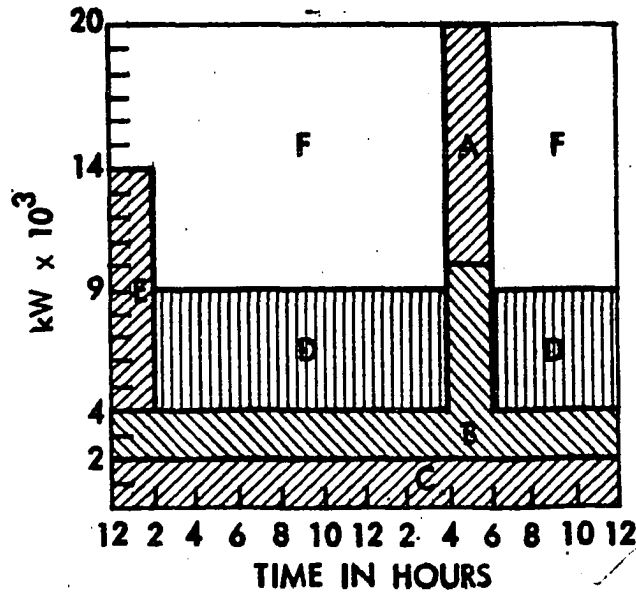


Figure A.1. Load duration curve

Assume that the total fixed charges for a 20,000 KW plant are \$100,000 per month.

1. By the energy method:

$$\text{Customer A: } 600,000 \times \frac{100,000}{7,440,000} = \$ 8,070 \text{ per month}$$

$$\text{Customer B: } 1,800,000 \times \frac{100,000}{7,440,000} = \$24,200 \text{ per month}$$

$$\text{Customer C: } 1,440,000 \times \frac{100,000}{7,440,000} = \$19,300 \text{ per month}$$

and so on.

2. By the peak responsibility method:

$$\text{Customer A: } 10,000 \times \frac{100,000}{20,000} = \$50,000 \text{ per month}$$

$$\text{Customer B: } 8,000 \times \frac{100,000}{20,000} = \$40,000 \text{ per month}$$

$$\text{Customer C: } 2,000 \times \frac{100,000}{20,000} = \$10,000 \text{ per month}$$

3. By the maximum demand method:

$$\text{Customer A: } 10,000 \times \frac{100,000}{35,000} = \$28,580 \text{ per month}$$

$$\text{Customer B: } 8,000 \times \frac{100,000}{35,000} = \$22,840 \text{ per month}$$

$$\text{Customer C: } 2,000 \times \frac{100,000}{35,000} = \$ 5,720 \text{ per month}$$

and so on.

4. By Greene's method:

$$D = 35,000 \text{ KW}$$

$$K = 7,440,000 \text{ KW-Hr/month}$$

$$C = \$100,000 \text{ per month}$$

$$P = 20,000 \text{ KW}$$

$$7,440,000 x + 35,000 y = 100,000$$

$$720 x + y = 5$$

$$x = \$0.00422 \text{ per KW-Hr}$$

$$y = \$1.96 \text{ per KW}$$

$$\begin{aligned} \text{Customer A: } & (600,000 \times 0.00422) + (10,000 \times 1.96) \\ & = \$22,140 \text{ per month} \end{aligned}$$

$$\begin{aligned} \text{Customer B: } & (1,800,000 \times 0.00422) + (8,000 \times 1.96) \\ & = \$23,290 \text{ per month} \end{aligned}$$

and so on.

5. By the Eisenmenger method:

$$\begin{aligned} F_A &= (0.5 \times 10,000) + (0.5 \times 10,000 \times \frac{2}{24}) + 0 \\ &= 5,000 + 417 \\ &= 5,417 \end{aligned}$$

$$\begin{aligned} F_B &= (8,000 \times \frac{8,000}{20,000}) + (8,000 \times \frac{12,000}{20,000} \times \frac{2}{24}) + (2,000 \times \frac{22}{24}) \\ &= 3,200 + 400 + 1,835 \\ &= 5,435 \end{aligned}$$

$$\begin{aligned} F_C &= (2,000 \times \frac{2,000}{20,000}) + (2,000 \times \frac{18,000}{20,000} \times \frac{2}{24}) + (2,000 \times \frac{22}{24}) \\ &= 200 + 150 + 1,835 \\ &= 2,185 \end{aligned}$$

$$F_D = 0 + 0 + (5,000 \times \frac{20}{24}) = 4,170$$

$$F_E = 0 + 0 + (10,000 \times \frac{2}{24}) = 844$$

$$\therefore F_A + F_B + F_C + F_D + F_E = 18,051$$

$$\text{Customer A: } \frac{5,417}{18,051} \times 100,000 = \$30,000 \text{ per month}$$

$$\text{Customer B: } \frac{5,435}{18,051} \times 100,000 = \$31,000 \text{ per month}$$

$$\text{Customer C: } \frac{2,185}{18,051} \times 100,000 = \$12,100 \text{ per month}$$

$$\text{Customer D: } \frac{4,170}{18,051} \times 100,000 = \$23,100 \text{ per month}$$

$$\text{Customer E: } \frac{844}{18,051} \times 100,000 = \$ 4,680 \text{ per month}$$

6. By the phantom method:

The average fixed charge under 100% load factor operation equals 0.694 cent per kilowatt-hour.

$$\$100,000 \div (20,000 \times 30 \times 24) = \$0.00694 = 0.694\text{¢}$$

Kilowatt-hours:

$$\text{Customer A: } 10,000 \times 2 \times 30 = 600,000 \text{ per month}$$

$$\begin{aligned} \text{Customer B: } & (2,000 \times 24 \times 30) + (6,000 \times 2 \times 30) \\ & = 1,800,000 \text{ per month} \end{aligned}$$

$$\text{Customer C: } 2,000 \times 24 \times 30 = 1,440,000 \text{ per month}$$

$$\text{Customer D: } 5,000 \times 20 \times 30 = 3,000,000 \text{ per month}$$

$$\text{Customer E: } 10,000 \times 2 \times 30 = 600,000 \text{ per month}$$

Total kilowatt-hours, all customers: 7,440,000 per month

Total kilowatt-hours at 100 load factor: 14,400,000 per month

Kilowatt-hours, phantom F: 6,960,000 per month

Demand charge to phantom F = $6,960,000 \times 0.00694 = \$48,250$ per month.

This amount is to be divided between customers A and B in proportion to their contribution to the peak.

The only reason there is a peak load is because these customers use more than their average demand at that particular instant. So their degree of contribution to the peak is limited to the excess demands over the average.

The average demand equals total consumption divided by total hours.

For Customer A:

$$\begin{aligned}\text{Average demand: } 600,000 \div (24 \times 30) &= 833 \text{ kW} \\ \text{Demand on peak:} &= 10,000 \text{ kW} \\ \text{Excess demand on peak:} &= 10,000 - 833 \\ &= 9,167 \text{ kW}\end{aligned}$$

For Customer B:

$$\begin{aligned}\text{Average demand: } 1,800,000 \div (24 \times 30) &= 2,500 \text{ kW} \\ \text{Demand on peak:} &= 8,000 \text{ kW} \\ \text{Excess demand on peak:} &= 5,500 \text{ kW} \\ \text{Total kilowatt load contributing to peak:} &= 9,167 + 5,500 \\ &= 14,667 \text{ kW} \\ \text{Demand charge of phantom F:} &= \$48,250\end{aligned}$$

$$\$48,250 \div 14,667 = \$3.29 \text{ per kW of demand in excess of average.}$$

Total demand charge:

$$\begin{aligned}\text{Customer A: } (3.29 \times 9,167) + (600,000 \times .00694) \\ &= \$34,410 \text{ per month} \\ \text{Customer B: } (3.29 \times 5,500) + (1,800,000 \times .00694) \\ &= \$30,600 \text{ per month} \\ \text{Customer C: } &(1,440,000 \times .00694) \\ &= \$10,000 \text{ per month} \\ \text{Customer D: } &(3,000,000 \times .00694) \\ &= \$20,820 \text{ per month} \\ \text{Customer E: } &(600,000 \times .00694) \\ &= \$4,170 \text{ per month}\end{aligned}$$

An equivalent statement of the phantom customer method is as follows (36):

The equivalent demands under the phantom customer method are the average demands for the period plus the excess of the system peak load over the total average demand spread over the individual items on the basis of excess of peak responsibility over average demand (positive value only).

From the above example the equivalent demand of Customer A

$$= \frac{600,000}{24 \times 30} + \frac{9,167}{14,667} \left(20,000 - \frac{7,440,000}{30 \times 24} \right) \text{ kW}$$

$$= 6,882 \text{ kW ,}$$

and the equivalent demand of Customer B

$$= \frac{1,800,000}{24 \times 30} + \frac{5,500}{14,667} \left(20,000 - \frac{7,440,000}{30 \times 24} \right) \text{ kW}$$

$$= 6,120 \text{ kW ,}$$

and so on.

7. By the weighted peak method

Weighted peak of Customer A = 10,000 kW

Weighted peak of Customer B = 8,000 kW

Weighted peak of Customer C = 2,000 kW

Weighted peak of Customer D = $0 + \left(\frac{9,000}{20,000} \times 5,000 \right) = 2,250 \text{ kW}$

Weighted peak of Customer E = $0 + \left(\frac{14,000}{20,000} \times 10,000 \right) = 7,000 \text{ kW}$

Total weighted peak = 29,250 kW

$$\begin{aligned}\text{Demand costs to A} &= \frac{10,000}{29,250} (10,000) = \$34,200 \\ \text{Demand costs to B} &= \frac{8,000}{29,250} (10,000) = \$27,400 \\ \text{Demand costs to C} &= \frac{2,000}{29,250} (10,000) = \$ 6,800\end{aligned}$$

etc.

8. Summary of results

The results from these methods will be compared in Table A.2.

Table A.2. Comparison of various methods

	kW-Hr	Peak Respon- sibility	Max. Demand	Greene's Theory	Eisen- menger's Theory	Phantom	Weighted Peak
A	\$ 8,070	\$50,000	\$28,580	\$22,140	\$30,000	\$34,410	\$34,200
B	24,200	40,000	22,840	23,290	31,000	30,600	27,400
C	19,360	10,000	5,720	10,000	12,100	10,000	6,800
D	40,300	0	14,280	22,430	23,100	20,820	7,700
E	8,070	0	28,580	22,140	4,680	4,170	23,900

Applying the average fixed costs of \$5 per kilowatt per month to the above table, we get the equivalent kilowatts allocated to each group of customers, which is shown in Table A.3.

Table A.3. Equivalent kilowatts allocated

	kW-Hr	Peak Responsi- bility	Max. Demand	Greene's Theory	Eisen- menger's Theory	Phantom	Weighted Peak
A	1,614	10,000	5,716	4,428	6,000	6,882	6,850
B	4,840	8,000	4,568	4,658	6,200	6,120	5,480
C	3,872	2,000	1,144	2,000	2,420	2,000	1,360
D	8,060	0	2,856	4,486	4,620	4,164	1,540
E	1,614	0	5,716	4,428	936	834	4,780

None of these methods is perfect. One method has some advantage and disadvantage over the others. Logically, the phantom method is believed to allocate the costs on an equitable basis. The peak customers who are responsible for the large investment have allocated to them the costs they incur. To the off-peak customers is allocated a cost to cover merely the energy used. The 100 per cent load factor customer does not benefit by the power plant service as much as do those customers with a poor load factor because he has no diversity with other customers and has to carry the entire fixed costs of the equipment used to serve him, where as other customers can share this cost between them. However, his fixed cost per unit of power demand is just as small as that of the off-peak customer.

From the above reasons the phantom method could be used to assign fixed costs on a monthly or daily basis as an equipment is actually used. This will be shown in Appendix C.

XIII. APPENDIX B: DERIVATION OF JACOBIAN MATRIX ELEMENTS

The coefficients of Equation 4.11, H_{km} , N_{km} , J_{km} , and L_{km} can be evaluated by taking partial derivatives of the real and reactive power as follows. First, rewrite Equation 4.10 as follows:

$$P_k + jQ_k = E_k e^{j\theta_k} \sum_{m=1}^N (E_m e^{-j\theta_m}) (Y_{km} e^{-j\phi_{km}}) \quad [B.1]$$

where k has some value between 1 and N . Take the partial derivative of Equation B.1 with respect to $\theta_m \neq \theta_k$.

$$\frac{\partial P_k}{\partial \theta_m} + j \frac{\partial Q_k}{\partial \theta_m} = -j(E_k e^{j\theta_k}) \times (E_m e^{-j\theta_m}) (Y_{km} e^{-j\phi_{km}}) \quad m \neq k \quad [B.2]$$

Define

$$\begin{aligned} a_m + jb_m &= E_m e^{j\theta_m} \times Y_{km} e^{j\phi_{km}} \\ &= (e_m + jf_m)(G_{km} + jB_{km}) \\ &= (e_m G_{km} - f_m B_{km}) + j(f_m G_{km} + e_m B_{km}) \\ \therefore a_m &= e_m G_{km} - f_m B_{km} \\ b_m &= f_m G_{km} + e_m B_{km} \\ (E_m e^{-j\theta_m}) (Y_{km} e^{-j\phi_{km}}) &= (e_m - jf_m)(G_{km} - jB_{km}) \\ &= (e_m G_{km} - f_m B_{km}) - j(f_m G_{km} + e_m B_{km}) \end{aligned}$$

The Equation B.2 can be rewritten in rectangular form as

$$\frac{\partial P_k}{\partial \theta_m} + j \frac{\partial Q_k}{\partial \theta_m} = -j(e_k + jf_k)(a_m - jb_m) \quad [B.3]$$

Equating real and imaginary parts on each side of Equation B.3 gives the following value for H_{km} and J_{km} when k is not equal to m .

$$\begin{aligned} H_{km} &= \frac{\partial P_k}{\partial \theta_m} = a_m f_k - b_m e_k \\ J_{km} &= \frac{\partial Q_k}{\partial \theta_m} = -(a_m e_k + b_m f_k) \end{aligned} \quad k \neq m \quad [B.4]$$

Take the partial derivative of Equation B.1 with respect to one value of E_m other than E_k , and then multiply and divided by E_m on the right-hand side.

$$\frac{\partial P_k}{\partial E_m} + j \frac{\partial Q_k}{\partial E_m} = \frac{1}{E_m} (E_k e^{j\theta_k}) (E_m e^{-j\theta_m}) (Y_{km} e^{-j\phi_{km}}) \quad m \neq k \quad [B.5]$$

Substituting as before for the last two terms, rewriting the equation in rectangular form, and equating the real and imaginary parts gives the following values of N_{km} and L_{km} when k is not equal to m .

$$\begin{aligned} N_{km} &= \frac{\partial P_k}{\partial E_m} = \frac{a_m e_k + b_m f_k}{E_m} \\ L_{km} &= \frac{\partial Q_k}{\partial E_m} = \frac{a_m f_k - b_m e_k}{E_m} \end{aligned} \quad m \neq k \quad [B.6]$$

To evaluate the coefficients when m equals k , a similar method can be used, except that in taking the derivatives the term in the summation where m is equal to k must be considered. First, take the partial derivative of Equation B.1 with respect to θ_k

$$\begin{aligned}
\frac{\partial P_k}{\partial \theta_k} + j \frac{\partial Q_k}{\partial \theta_k} &= j [E_k e^{j\theta_k}] \left[\sum_{m=1}^N (E_m e^{-j\theta_m}) (Y_{km} e^{-j\phi_{km}}) \right] \\
&\quad - j (E_k e^{j\theta_k}) (E_k e^{-j\theta_k}) (Y_{kk} e^{-j\phi_{kk}}) \\
&= j(P_k + jQ_k) - jE_k^2 (G_{kk} - jB_{kk}) \quad [B.7]
\end{aligned}$$

Then

$$\begin{aligned}
H_{kk} &= \frac{\partial P_k}{\partial \theta_k} = -Q_k - B_{kk} E_k^2 \\
J_{kk} &= \frac{\partial Q_k}{\partial \theta_k} = P_k - G_{kk} E_k^2 \quad m = k \quad [B.8]
\end{aligned}$$

In a manner similar to that used to obtain Equation B.7, the partial derivative of Equation B.1 can be taken with respect to E_k .

Using the same substitutions as for Equation B.7 yields

$$\frac{\partial P_k}{\partial E_k} + j \frac{\partial Q_k}{\partial E_k} = \frac{1}{E_k} (P_k + jQ_k) + E_k (G_{kk} - jB_{kk})$$

Then

$$\begin{aligned}
N_{kk} &= \frac{\partial P_k}{\partial E_k} = \frac{P_k}{E_k} + G_{kk} E_k \\
L_{kk} &= \frac{\partial Q_k}{\partial E_k} = \frac{Q_k}{E_k} - B_{kk} E_k \quad m = k \quad [B.9]
\end{aligned}$$

This completes the derivation for all terms in the Jacobian matrix.

XIV. APPENDIX C: NUMERICAL EXAMPLES OF SENSITIVITY COMPUTATION

The proposed method of sensitivity computation is applied to the six-bus system (Figure C.1) borrowed from Ward and Hale (37). Table C.1 contains the positive sequence line impedances in p.u. Table C.2 contains the nodal admittance matrix. The operating condition of the system at an average load level is shown in Figure C.1. This figure also shows an arbitrary division of the system into two parts called Area 1 and Area 2 which will be considered as areas of different ownership.

Table C.1. Line impedances (p.u.)

Bus to bus	R	X
1-4	0.08	0.37
1-6	0.123	0.518
2-3	0.723	1.05
2-5	0.282	0.64
3-4	0	0.133
4-6	0.097	0.407
5-6	0	0.30

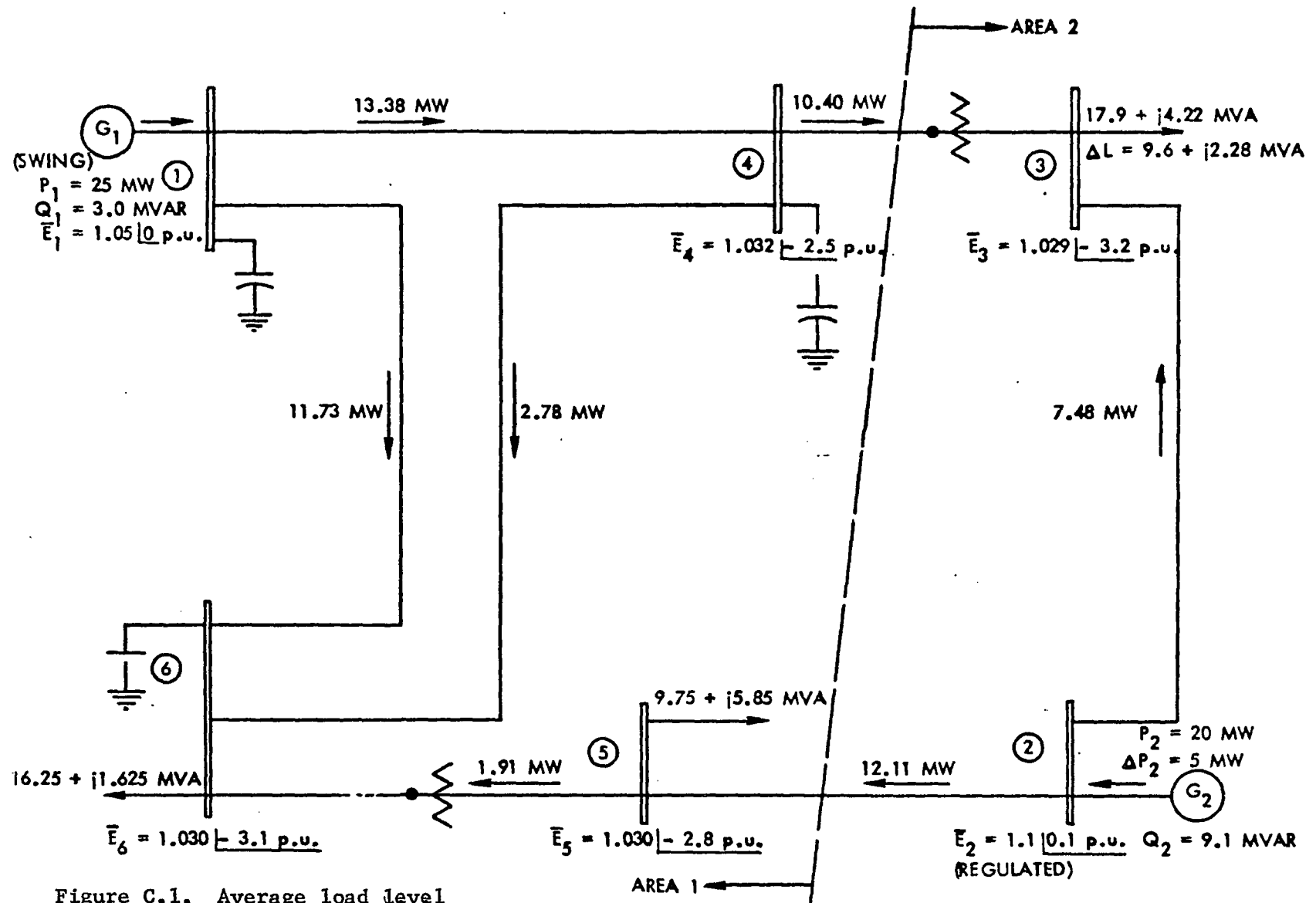


Table C.2. Nodal admittance matrix (p.u.)

k-m	G_{km}	B_{km}
1-1	0.9922	-4.401
1-4	-0.5583	2.582
1-6	-0.4339	1.8275
2-2	1.0214	-1.9546
2-3	-0.449	0.6461
2-5	-0.5765	1.3085
3-3	0.449	-8.1649
3-4	0	7.5188
4-4	1.1124	-12.4257
4-6	-0.5541	2.3249
5-5	0.5765	-4.6418
5-6	0	3.3333
6-6	0.988	-7.4857

It is assumed that

1. The system represents a power pool with 2 member companies (Area 1 and 2). There are 2 buses (2 and 3) in Area 2 and 4 buses (1, 4, 5, and 6) in Area 1.
2. Bus 1 in Area 1 is the swing bus of the pool.
3. Bus 2 in Area 2 is a regulated bus with sufficient reactive power to keep E_2 constant.

4. P_2 is an economically scheduled generator.
5. Power into the system has a positive sign and power out of the system (loads) has a negative sign.
6. An increment (Δ) means a quantity at a new condition minus the quantity at an old condition.

It is now assumed that the load at bus 3 is increased and the generation at bus 2 is increased with all other loads constant. In particular let

$$\Delta P_3 = -9.6 \text{ MW} = -0.096 \text{ p.u.}$$

$$\Delta Q_3 = -2.28 \text{ MW} = -0.0228 \text{ p.u.}$$

$$\Delta P_2 = 5 \text{ MW} = 0.05 \text{ p.u.}$$

Note that the increase in scheduled generation at bus 2 is not as great as the increase in load at bus 3. The operating condition of the system at an assumed peak load level is shown in Figure C.2. The modified sensitivity matrices defined in Equations 4.27, 4.28, and 4.29 are shown in Tables C.3, C.4, and C.5, respectively. They are evaluated at the average condition. Following the changes ΔP_3 , ΔQ_3 , and ΔP_2 assumed above, Equations 4.27, 4.28, and 4.29 will be solved and the results will be compared with the results obtained from solving the ordinary load flow problem using a conventional method (ac model method). Comparison of the results is shown in Table C.6. Column 2 (ac model method) and Column 5 (Jacobian method) show very close results because the Jacobian method is actually the conventional Newton's method with

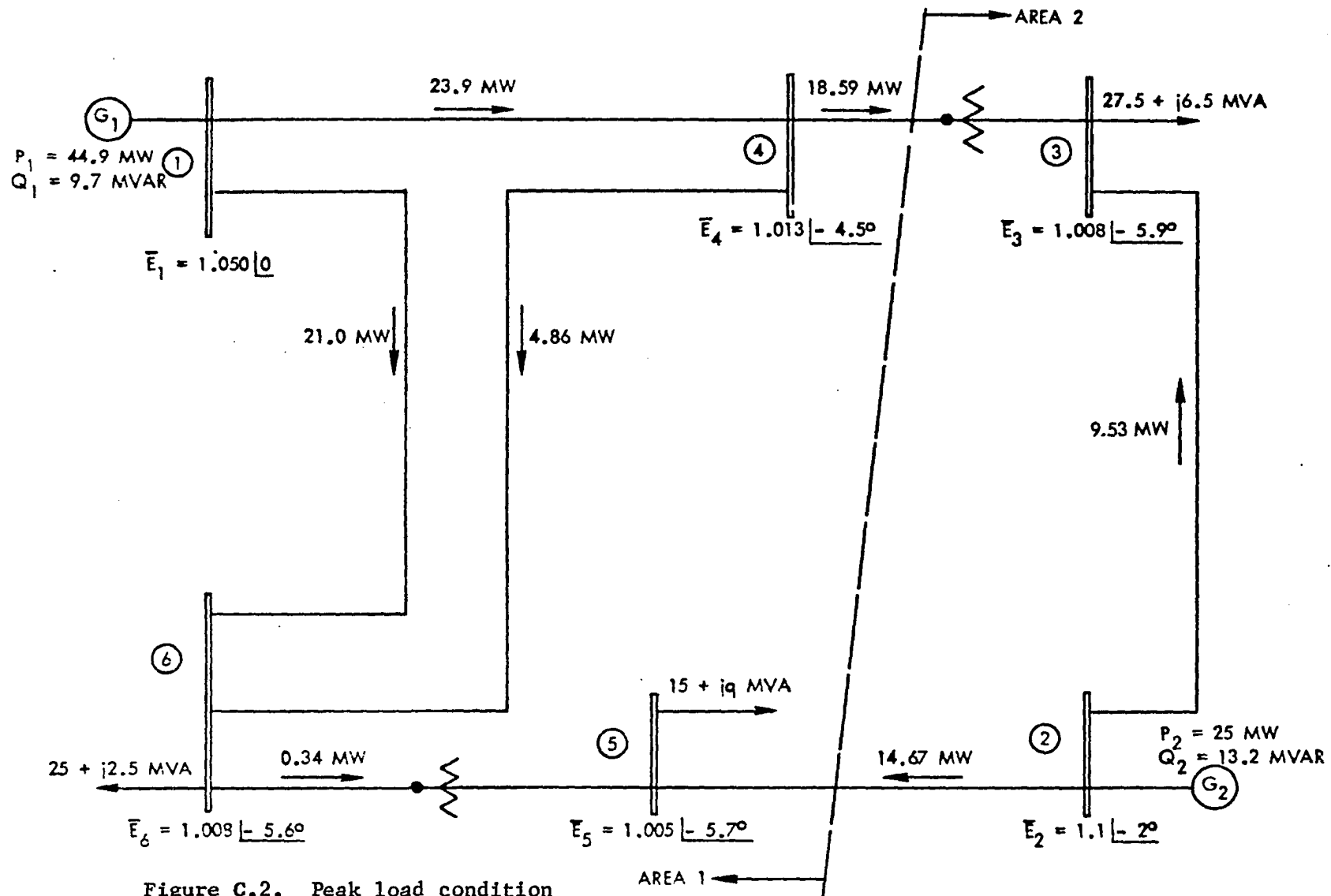


Table C.3. The modified sensitivity matrix (Jacobian matrix inverted)

	$(\Delta\theta_1)^a$	ΔP_2	ΔP_3	ΔP_4	ΔP_5	ΔP_6	$(\Delta E_1)^a$	$(\Delta E_2)^a$	ΔQ_3	ΔQ_4	ΔQ_5	ΔQ_6
ΔP_1	-1.000	-0.954	-1.023	-1.022	-1.019	-1.023	0.000	0.000	-0.016	-0.013	-0.020	-0.014
$\Delta\theta_2$	0.000	0.743	0.256	0.203	0.410	0.247	0.000	0.000	0.066	0.040	0.066	0.028
$\Delta\theta_3$	0.000	0.232	0.352	0.240	0.181	0.156	0.000	0.000	-0.034	-0.034	0.000	-0.011
$\Delta\theta_4$	0.000	0.185	0.237	0.244	0.161	0.148	0.000	0.000	-0.035	-0.037	-0.004	-0.013
$\Delta\theta_5$	0.000	0.385	0.185	0.166	0.505	0.270	0.000	0.000	0.014	0.002	-0.031	-0.033
$\Delta\theta_6$	0.000	0.228	0.157	0.150	0.264	0.287	0.000	0.000	0.000	-0.005	-0.026	-0.035
ΔQ_1	0.000	0.310	-0.061	-0.032	0.023	-0.008	-1.000	0.000	-0.677	-0.750	-0.474	-0.700
ΔQ_2	0.000	-0.287	-0.061	-0.065	-0.131	-0.111	0.000	-1.000	-0.361	-0.280	-0.576	-0.337
ΔE_3	0.000	-0.064	0.067	0.054	-0.001	0.016	0.000	0.000	0.282	0.182	0.052	0.082
ΔE_4	0.000	-0.034	0.034	0.055	0.015	0.023	0.000	0.000	0.183	0.202	0.061	0.092
ΔE_5	0.000	-0.072	0.011	0.013	0.055	0.048	0.000	0.000	0.052	0.061	0.308	0.152
ΔE_6	0.000	-0.227	0.030	0.027	0.058	0.061	0.000	0.000	0.083	0.092	0.152	0.222

^aThe quantities in parentheses are scheduled to be zero.

only one iteration. The results shown in Column 3 and Column 4 show a larger error because constant voltages have been assumed.

When the state of the system is changed, power flows in the lines must change accordingly. The increment of line power flows are compared in Table C.7. An increment of power flow is defined as the magnitude of the flow at a new system state minus the magnitude of the flow at a previous system state. A negative quantity of the increment of power flow in a line indicates that the power flow is decreased in that line when the system state is changed to a new state.

For example, when the sample system is operating at the average load level, power flow in the line connecting bus 5 and bus 6 is 1.91 MW. When the system is operating at the peak load level, power flow in the line is 0.34 MW. The increment of line power flow in this particular case is -1.57 MW (See Table G.1, page 141).

The computer program used to compute the tables of Appendix C will be shown in Appendix D.

Table C.4. The modified sensitivity matrix H_{km}^{-1} (approximation)

	$(\Delta E_1)^a$	ΔP_2	ΔP_3	ΔP_4	ΔP_5	ΔP_6
ΔP_1	-1.000	-0.961	-1.020	-1.018	-1.016	-1.020
$\Delta \theta_2$	0.000	0.772	0.243	0.191	0.404	0.240
$\Delta \theta_3$	0.000	0.228	0.364	0.249	0.181	0.158
$\Delta \theta_4$	0.000	0.180	0.250	0.254	0.162	0.151
$\Delta \theta_5$	0.000	0.385	0.183	0.163	0.517	0.280
$\Delta \theta_6$	0.000	0.227	0.159	0.151	0.280	0.297

^aThe quantity in parentheses is scheduled to be zero.

Table C.5. The modified sensitivity matrix $\underline{\gamma}^{-1}$ from dc power flow model

	$(\Delta E_1)^a$	ΔP_2	ΔP_3	ΔP_4	ΔP_5	ΔP_6
ΔP_1	-1	-1	-1	-1	-1	-1
$\Delta \theta_2$	0	0.8062	0.241	0.189	0.406	0.239
$\Delta \theta_3$	0	0.2408	0.362	0.247	0.182	0.157
$\Delta \theta_4$	0	0.1893	0.247	0.252	0.161	0.150
$\Delta \theta_5$	0	0.4060	0.182	0.161	0.515	0.278
$\Delta \theta_6$	0	0.2390	0.157	0.150	0.278	0.295

^aThe quantity in parentheses is scheduled to be zero.

As discussed in Chapter 3, the distribution benefit of a transmission line relates only to loads. If a radial line is serving a single load, the distribution benefit of the line to that load is indicated by the power flow to that load. In a large transmission system the transmission lines are connected in loops and there is no easy way to tell that the load is receiving power from a particular line. In other words, there is no easy way to convince people that their loads are served by a given line or lines.

One way to solve this problem is to study the sensitivity of the changes of loads on the power flows. As a load changes from one level to another level the magnitude of transmission line power flow also changes. This means that the additional power is distributed to those

Table C.6. Comparison of the results (load increased in Area 2)

	ac model ^a method	dc model ^b method	Approx. ^c method	Jacobian ^d method
ΔP_1	5.3 MW	4.6 MW	4.987 MW	5.096 MW
$\Delta \theta_2$	0.6°	0.983°	0.8737°	0.631°
$\Delta \theta_3$	-1.3°	-1.3°	-1.349°	-1.23°
$\Delta \theta_4$	-0.7°	-0.817°	-0.855°	-0.729°
$\Delta \theta_5$	0°	-0.182°	0.096°	0.0676°
$\Delta \theta_6$	-0.2°	-0.179	-0.222	-0.2114°
ΔQ_1	3.9 MVAR			3.678 MVAR
ΔQ_2	0.1 MVAR	not	not	-0.022 MVAR
ΔE_3	-0.017 p.u.	available	available	-0.016 p.u.
ΔE_4	-0.012 p.u.			-0.012 p.u.
ΔE_5	-0.006 p.u.			-0.0058 p.u.
ΔE_6	-0.006 p.u.			-0.0059 p.u.

^aConventional load flow analysis.

^bEquation 4.29, the dc model method.

^cEquation 4.28, the approximate method.

^dEquation 4.27, the Jacobian method.

Table C.7. Comparison of increment of line flows (MW) (load increased in Area 2)

	ac model method	dc model method	Approx. method	Jacobian method
ΔP_{1-4}	4.17	4.000	4.217	4.114
ΔP_{1-6}	0.99	0.615	0.776	0.941
ΔP_{2-3}	2.86	2.880	2.978	3.170
ΔP_{2-5}	1.80	2.120	2.059	1.808
ΔP_{3-4}	6.74	6.700	6.875	6.522
ΔP_{4-6}	-2.63	-2.760	-2.733	-1.224
ΔP_{5-6}	1.68	2.100	1.964	1.680

lines to serve the change in load. That additional power flow will be used to indicate the distribution benefits of those lines to the loads. Any of the four methods described above could be used as a basis to compute the distribution benefit. However, it takes more time and costs more money to solve power flow equations which are nonlinear. That is why the dc model of power flow equations are often used in long-range transmission planning which does not require great accuracy.

When the load of the Area 2 is increased from one load level (average) to the other level (peak) and all the loads in the Area 1

are unchanged, the changes of power flows in the transmission lines are shown in Table C.7. These changes are obtained by 4 different methods. The results shown in Column 2 and Column 3 are of the most interest because they give a good comparison between the ac model method and the dc model method. These results indicate the distribution benefit of all lines to the load in Area 2, measured in MW.

Tables C.8 and C.9 show similar results as in Tables C.6 and C.7 except that the results are obtained when the loads of the Area 1 are increased from one load level (average) to the other load level (peak) and the load in Area 2 is not changed (average level). These results indicate the distribution benefit of those lines to the loads in Area 1, measured in MW.

Table C.8. Comparison of results (load increased in Area 1)

	ac model method	dc model method
ΔP_1	14.8 MW	14 MW
$\Delta \theta_2$	-2.7°	-2.45°
$\Delta \theta_3$	-1.4°	-1.335°
$\Delta \theta_4$	-1.2°	-1.234°
$\Delta \theta_5$	-2.9°	-2.94°
$\Delta \theta_6$	-2.3°	-2.3°

Table C.9. Comparison of increment of line flows (load increased in Area 1)

	ac model method	dc model method
ΔP_{1-4}	6.30 MW	6.04 MW
ΔP_{1-6}	8.35 MW	7.926 MW
ΔP_{2-3}	-1.41 MW	-1.364 MW
ΔP_{2-5}	0.82 MW	1.352 MW
ΔP_{3-4}	1.43 MW	1.3667 MW
ΔP_{4-6}	5.47	4.64 MW
ΔP_{5-6}	0.08	0.135 MW

XV. APPENDIX D: COMPUTER PROGRAMS

The computer program used to compute the tables of Appendix C will be shown in this Appendix. The program is not as efficient as it might be because it was developed just to facilitate the computations. It can handle only a six bus system or smaller system. For a bigger system the program needs changes in the dimension statements.

The program will compute all coefficients in the Jacobian matrix, using Equations 4.12 and 4.13. With a little adjustment the program can compute only the coefficients H_{km} and H_{kk} for the approximate method. Then the matrix will be modified to assume the matrix equation shown in Equations 4.27 and 4.28. The next part of the program calculates a new state of the system when there is a load change, i.e., all data in the far right column of Equations 4.27 or 4.28 are read into the program. The last part of the program computes the difference in magnitude of the line flow at the new state and the line flow at the previous state to obtain the distribution benefit.

The program is not developed to handle the dc model method. All the calculation in the dc model method, Equation 4.29, were performed on an electronic desk calculator except for the matrix-inversion part of the calculation.


```

      A(M)= G(K,M)*EM(M)-ZETA(K,M)*FM(M)
      DELTA(M)= G(K,M)*FM(M) + ZETA(K,M)*EM(M)
      IF(K.EQ.M) GO TO 40
C    THIS PART CALCULATES A JACOBIAN MATRIX
      Y(K,M)      = A(M)*FM(K) - DELTA(M)*EM(K)
      Z(K,M)      = (A(M)*EM(K) + DELTA(M)*FM(K))/E(M)
      AJ(K,M)     = -Z(K,M)*E(M)
      AL(K,M)     = Y(K,M)/E(M)
      GO TO 35
40   Y(K,K)      = -Q(K)-ZETA(K,K)*E(K)*E(K)
      Z(K,K)      = P(K)/E(K) + G(K,K)*E(K)
      AJ(K,K)     = P(K)-G(K,K)*E(K)*E(K)
      AL(K,K)     = Q(K)/E(K) - ZETA(K,K)*E(K)
35   CONTINUE
45   CONTINUE
      WRITE (3,125)
125  FORMAT('1','THE JACOBIAN MATRIX')
      NN=N+1
      DO 50 K=1,N
      WRITE (3,110) (Y(K,M),M=1,N ), (Z(K,M),M=1,N )
110  FORMAT (1H ,12(F10.5))
      50 CONTINUE
      DO 55 K=1,N
      WRITE(3,110) (AJ(K,M),M=1,N ), (AL(K,M),M=1,N )
55   CCNTINUE
      NN2 =  N*2
      DO 60 K=1,N
      DO 60 M=NN,NN2
60   Y(K,M) = Z(K,M-N )
      DO 65 K=NN,NN2
      DO 65 M=1,N
      Y(K,M) = AJ(K-N ,M)
65   Y(K,M+N ) = AL(K-N ,M)
      CALL MATINV(Y,NN2)
      WRITE (3,120)
120  FORMAT('0','MODIFIED SENSITIVITY MATRIX')
C    THIS PART CALCULATES THE MODIFIED SENSITIVITY MATRIX

```

```

      I=1
185 COF = -Y(I,I)
      DO 177 J=1,NN2
177 Y(I,J) = Y(I,J)/COF
      DO 180 K = 1,NN2
      IF(K.EQ.I) GO TO 180
      COF = Y(K,I)
      DO 175 J=1,NN2
175 Y(K,J) = Y(K,J)+Y(I,J)*COF
180 CONTINUE
      IF(I.GE.NN) GO TO 176
      I=NN
      GO TO 185
176 IF(I.GE.(N+REG)) GO TO 184
      I= N+REG
      GO TO 185
184 DO 5 I=1,NN2
      WRITE (3,110) (Y(I,J),J=1,NN2)
      5 CONTINUE

```

```

C
C THIS PART DETERMINES A NEW STATE OF THE SYSTEM
C P(1)=Q(1)=Q(REG)=0.0
C P,Q INCREMENTAL NET POWER PU
C

```

```

      READ (1,100) (P(K),K=1,N ), (Q(K),K=1,N )
      DO 75 K=1,NN2
      IF(K.GT.N ) GO TO 76
      DELTA(K)=P(K)
      GO TO 75
76 DELTA(K)=Q(K-N )
75 CCNTINUE
      DO 7 I=1,NN2
      A(I)=0.0
      DO 7 J=1,NN2
      A(I)= A(I)+Y(I,J)*DELTA(J)
7 CONTINUE
      WRITE (3,150)

```

```

150 FORMAT('0','INCREMENT MATRIX OF ANGLES AND VOLTAGES')
    DO 80 K=1,NN2
        WRITE (3,160) A(K)
160  FORMAT(' ',F10.5)
    80  CONTINUE
C
C    CALCULATE INCREMENTAL POWER FLOW FROM BUS I TO BUS J
C
C    INCLUDE SWING BUS ANGLE CHANGE IN ARRAY A
    A(1)=0.0
    A(N+1)=0.0
    A(N+REG)=0.0
    WRITE (3,250)
250  FORMAT('0',10X,'FROM BUS',4X,'TO BUS',8X,'INCREMENT OF POWER FLOW'
1)
    DO 205 I=1,N
        DO 206 J=1,N
            IF(J.EQ.I) GO TO 206
            IF((G(I,J).EQ.0.0).AND.(ZETA(I,J).EQ.0.0)) GO TO 206
            DUM1 = ZIGMA(I)-ZIGMA(J)
            DUM2 = E(I)*E(J)
            DUM3 = G(I,J)*DCOS(DUM1) + ZETA(I,J)*DSIN(DUM1)
            PIJ1 = DUM2*DUM3
            DUM1 = DUM1 + (A(I)-A(J))
            DUM2 = (E(I)+A(I+N))*(E(J)+A(J+N))
            DUM3= G(I,J)*DCOS(DUM1) + ZETA(I,J)*DSIN(DUM1)
            PIJ2 = DUM2*DUM3
            PIJ2 = ABS(PIJ2)-ABS(PIJ1)
            WRITE (3,251) I,J,PIJ2
251  FORMAT(' ',15X,I2,9X,I2,15X,F10.5)
206  CONTINUE
205  CONTINUE
    STOP
    END
    SUBROUTINE MATINV ( B ,NN)
C
C    'CLEANED UP' GOING DOWN DIAGONAL BY LARGEST TERM ON DIAGONAL WIT&

```


C
C

DOUBLE PRECISION SUBTRACTION

```
DIMENSION B(12,12)
DIMENSION KCLICK(12)
DOUBLE PRECISION DINIJ, DINKJ, DINIK, DUMMY, ACCURT
DOUBLE PRECISION INOUT (12,12)
DOUBLE PRECISION A
DO 1 I=1,NN
DO 1 J=1,NN
1 INOUT(I,J)=B(I,J)
DO 10 INDEX = 1, NN
ACCURT = 0.0
DO 9 K = 1, NN
KL = K
DO 7 KLOP = 1, INDEX
IF(KLOP .EQ. INDEX) GO TO 7
IF( K .EQ. KCLICK(KLOP)) GO TO 9
7 CONTINUE
A = INOUT(K, K)
IF(A*A-ACCURT*ACCURT) 9,9,8
8 ACCURT = A
KICK = K
KCLICK(INDEX) = K
9 CONTINUE
DUMMY = 1.000 / ACCURT
DO 11 I = 1,NN
DO 11 J = 1,NN
IF( I .EQ. KICK .OR. J .EQ. KICK) GO TO 11
DINIJ = INOUT(I,J)
DINKJ = INOUT(KICK,J)
DINIK = INOUT(I,KICK)
INOUT(I,J) = (ACCURT * DINIJ - DINKJ * DINIK) * DUMMY
11 CONTINUE
DO 12 J = 1, NN
IF (J .EQ. KICK ) GO TO 12
DINKJ = INOUT(KCLICK(INDEX),J)
INOUT(KCLICK(INDEX),J) = -DINKJ * DUMMY
```

```

12 CONTINUE
  DO 13 I = 1, NN
    IF ( I .EQ. KICK ) GO TO 13
    DINIK = INOUT(I,KICK)
    INOUT( I, KICK ) = -DINIK * DUMMY
13 CONTINUE
  INOUT (KICK,      KICK) = - DUMMY
10 CONTINUE
  DO 14 I = 1, NN
    DO 14 J = 1, NN
14      B (I,J) = - INOUT (I,J)
  RETURN
  END

```

XVI. APPENDIX E: RELIABILITY FUNCTION (33)

Define

N_o = Fixed number of components at the beginning of the tests.

N_s = Number of surviving components at time t .

N_f = Number of components which have failed at time t .

The reliability or probability of survival is at any time t during the test given by

$$R(t) = \frac{N_s}{N_o} = \frac{N_s}{N_s + N_f} \quad [E.1]$$

where N_s and N_f are counted at that specific time t .

We can also define the probability of failure Q (called unreliability)

as

$$Q(t) = \frac{N_f}{N_o} = \frac{N_f}{N_s + N_f} \quad [E.2]$$

Then at any time t

$$R(t) + Q(t) = 1.$$

Also, reliability can be written as

$$R(t) = \frac{N_s}{N_s + N_f} = \frac{N_o - N_f}{N_o} = 1 - \frac{N_f}{N_o}.$$

By differentiation of this equation we obtain

$$\frac{dR(t)}{dt} = \frac{d(1 - N_f/N_o)}{dt} = -\frac{1}{N_o} \frac{dN_f}{dt} \quad [E.3]$$

Then

$$\frac{dN_f}{dt} = - N_o \frac{dR(t)}{dt}$$

which is the rate at which components fail and is equal to

$$\frac{dN_f}{dt} = \frac{d}{dt} (N_o - N_s) = - \frac{dN_s}{dt} .$$

Thus, this quantity is also the negative rate at which the components survive. The term dN_f can be interpreted as the number of components failing in the time interval dt between the times t and $t+dt$.

At the time t we still have N_s components in test; therefore $\frac{dN_f}{dt}$ components will fail out of these N_s components. If we divide both sides of the above equation by N_s , we obtain on the left the rate of failure or the instantaneous probability of failure per one component, which we call the failure rate λ :

$$\lambda = \frac{1}{N_s} \frac{dN_f}{dt} = - \frac{N_o}{N_s} \frac{dR(t)}{dt} \quad [E.4]$$

$$\lambda = \frac{-1}{R(t)} \frac{dR(t)}{dt}$$

which is the most general expression for the failure rate. In the general case, λ is a function of the operating time t . Only in one case will the equation yield a constant, and that is when failures occur exponentially at random intervals in time.

By rearrangement and integration, we obtain the general formula for reliability.

$$\lambda dt = - \frac{dR}{R}$$

$$\int_0^t \lambda dt = - \int_1^R \frac{dR}{R} = - \ln R$$

$$\ln R = - \int_0^t \lambda dt$$

Solving for R and knowing that at $t=0$, $R=1$, we obtain

$$R(t) = \exp \left[- \int_0^t \lambda dt \right] \quad [E.5]$$

When we specify that λ is constant, the exponent becomes

$$- \int_0^t \lambda dt = - \lambda t$$

and the reliability or the probability of a component surviving at time t in a constant failure rate environment is given by

$$R(t) = e^{-\lambda t} \quad [E.6]$$

From Equation E.3

$$\frac{dR(t)}{dt} = \frac{-1}{N_0} \frac{dN_f}{dt}$$

As $dt \rightarrow 0$, $\frac{1}{N_0} \frac{dN_f}{dt}$ is the instantaneous failure density $f(t)$. Then

$$f(t) = - \frac{dR(t)}{dt} = \lambda e^{-\lambda t} \quad [E.7]$$

XVII. APPENDIX F: RELIABILITY CALCULATIONS

The application of conditional probability to reliability computation is described in this appendix. This approach is applied to the system shown in Figure 6.2. The forced outage parameters of the transmission lines and the transformers are shown in Table F.1. The failure rates and the mean repair times of the components, defined in Chapter V, have been chosen from reference 38 and a report of the Electric Edison Institute (EEI).

Tables F.2 and F.3 show the calculated probability of the system in state B_j , that is $P(B_j)$. Table F.4 shows the load levels for different probability values which were taken from the assumed load duration curves.

The probability of system failure at bus k is shown in Tables F.5 and F.6.

Most of the computations were performed on an electronic desk calculator except the determination of the maximum load that can be supplied at a load bus before the load bus experiences difficulty (low voltage) due to the transmission outages. This was done by digital load flow analysis.

Samples of calculations

1. Table F.1:

At path 1 which is a transmission line,

Failure rate λ = 3 failures/year

Mean repair time r = 5.85 hours .

Then

$$\begin{aligned}\text{Repair rate } \mu &= \frac{8760}{5.85} \text{ repairs/year} \\ &= 1500 \text{ repairs/year} .\end{aligned}$$

From Equations 5.9 and 5.10

$$\text{Unavailability, } U = \frac{\lambda}{\lambda + \mu} = \frac{3}{3 + 1500} = 0.002$$

$$\text{Availability, } A = \frac{\mu}{\mu + \lambda} = \frac{1500}{3 + 1500} = 0.998 .$$

2. Table F.2:

Each transmission path state is defined by the path number on outage and all other paths are presumed in service. For example: Path number on outage (1, 2) means that in this transmission path state, path 1 and path 2 are out of service and the remaining paths (3, 4, 5, 6, 7) are in service. This corresponds to state number 3 ($B_j = 3$).

Path number on outage (none) means all paths are in service.

To calculate the probability that the system will be in the state 3, i.e., ($B_j = 3$), Equation 5.11 is used.

$$\begin{aligned}P(B_j) &= (A_1 \cdot A_2 \cdot \dots \text{ for } M \text{ components in}) \\ &\quad \cdot (U_1 \cdot U_2 \cdot \dots \text{ for } N-M \text{ components out})\end{aligned}$$

Then

$$\begin{aligned}P(B_j = 3) &= (A_3 \cdot A_4 \cdot A_5 \cdot A_6 \cdot A_7) \cdot (U_1 \cdot U_2) \\ &= (.998 \times .995 \times .697 \times .697 \times .9955) \times (0.002 \times 0.002) \\ &= .000001921 .\end{aligned}$$

3. Table F.3:

All the calculations are made in the same procedure as in Table F.2.

4. Table F.4:

Table F.4 is derived from Figure 6.3.

5. Table F.5:

(a) $P(B_j)$ is obtained from Table F.2.

(b) As discussed in Chapter 6, the load duration curve is divided into 6 steps at 0.2, 0.4, 0.6, 0.8, 0.9, and 1.0. Under each outage state B_j , if the voltage at load bus k exceeds the range between the minimum (0.95 p.u.) and maximum (1.1) acceptable values at any of the increasing load levels, $P_{LK}(B_j)$ is taken as the average of the duration of the load level at which the load bus fails and the previous load level. For example, consider outage (1, 2):

At the load level corresponding to duration = 1.0, the load buses are able to serve their load, but at the load level corresponding to duration = 0.9 the load buses fail to serve their load. We take the average value of 1.0 and 0.9 to get 0.95, that is

$$P_{LK}(B_j) = 0.95 .$$

If the load buses are able to serve their load even at the load level corresponding to duration 0.0 (maximum load), the probability will take the value 0.0.

(c) Equation 6.14 is used to calculate the last three columns of the table assuming 100% availability of the generators.

(d) The same procedure applies to Table F.6.

Table F.1. Forced outage parameters

Component	Path No.	<u>Failure rate</u> Number of failure/year (λ)	Mean repair time (r)	<u>Repair rate</u> Number of repairs per year (μ)	U Unavail-ability $\frac{\lambda}{\lambda + \mu}$	A Avail-ability $\frac{\mu}{\lambda + \mu}$
Transmission line	(1)	3 ^a	5.85 hours ^a	$\frac{8760}{5.85} = 1500$.002	.998
	(2)	3	5.85 hours	1500	.002	.998
	(3)	3	5.85 hours	1500	.002	.998
	(4)	1.5 ^a	2.98 hours ^a	3000	.005	.995
	(7)	4.5 ^a	8.76 hours ^a	1000	.0045	.9955
Transformer	(5)	2 ^b	1900 hours ^b	$\frac{8760}{1900} = 4.6$.303	.697
	(6)	2	1900 hours	4.6	.303	.697

^aThe data are estimated from an Electric Edison Institute Report on Transmission Reliability.

^bThe data are taken from Reference 38.

Table F.2. Transmission path states (Maximum number of paths = 7)

State no. B_j	Path number on outage	Number of outage	Probability $P(B_j)$
1	(none)	0	.478323287
2	(1)	1	.000958564
3	(1, 2)	2	.000001921
4	(1, 3)	2	.000001921
5	(1, 4)	2	.000004817
6	(1, 5)	2	.000416707
7	(1, 6)	2	.000416707
8	(1, 7)	2	.000004333
9	(2)	1	.000958564
10	(2, 3)	2	.000001921
11	(2, 4)	2	.000004817
12	(2, 5)	2	.000416706
13	(2, 6)	2	.000416706
14	(2, 7)	2	.000004333
15	(3)	1	.000958564
16	(3, 4)	2	.000004817
17	(3, 5)	2	.000416706
18	(3, 6)	2	.000416706
19	(3, 7)	2	.000004333
20	(4)	1	.002403635
21	(4, 5)	2	.001044909
22	(4, 6)	2	.001044909
23	(4, 7)	2	.000010865
24	(5)	1	.207936810
25	(5, 6)	2	.090394338
26	(5, 7)	2	.000939945
27	(6)	1	.207936810
28	(6, 7)	2	.000939945
29	(7)	1	.002162185

Table F.3. Transmission path states

Maximum number of paths = 6 (Path 7 - out)

State no. B_j	Path number on outage	Number of outage	Probability $P(B_j)$
1	(none)	0	.480485472
2	(1)	1	.000962897
3	(1, 2)	2	.000001930
4	(1, 3)	2	.000001930
5	(1, 4)	2	.000004839
6	(1, 5)	2	.000418591
7	(1, 6)	2	.000418591
8	(2)	1	.000962897
9	(2, 3)	2	.000001930
10	(2, 4)	2	.000004839
11	(2, 5)	2	.000418591
12	(2, 6)	2	.000418591
13	(3)	1	.000962897
14	(3, 4)	2	.000004839
15	(3, 5)	2	.000418591
16	(3, 6)	2	.000418591
17	(4)	1	.002414500
18	(4, 5)	2	.001049632
19	(4, 6)	2	.001049632
20	(5)	1	.208876755
21	(5, 6)	2	.090802951
22	(6)	1	.208876755

Table F.4. Load levels for different probability values

Load level, MW			Total MW	Probability of load exceeding the indicated load level
Bus 3	Bus 5	Bus 6		$P_{LK}(B_j)$
11.55	5.7	8.75	26.0	1.0
12.4	6.15	9.75	28.8	0.95
13.2	6.6	10.5	30.3	0.9
14.8	7.65	12	34.45	0.8
17.9	9.45	15.25	42.6	0.6
21.2	11.25	18.5	51.0	0.4
24.4	13.0	21.7	59.1	0.2
27.5	15.0	25.0	67.5	0.0

Table F.5. Reliability study (7 paths)

Path number on outage	Number of outage	$P(B_j)$	$P_{LK}(B_j)$			$P(B_j) \times P_{LK}(B_j)$		
			Bus 6	Bus 3	Bus 5	Bus 6	Bus 3	Bus 5
(none)	0	.478323287	0.0	0.0	0.0	0.0	0.0	0.0
(1)	1	.000958564	0.1	0.1	0.1	.0000958564	.0000958564	.0000958564
(1, 2)	2	.000001921	0.95	0.95	0.95	.000001825	.000001825	.000001825
(1, 3)	2	.000001921	0.7	0.7	0.7	.000001345	.000001345	.000001345
(1, 4)	2	.000004817	0.9	0.9	0.9	.000004335	.000004335	.000004335
(1, 5)	2	.000416707	0.9	0.9	0.9	.000375036	.000375036	.000375036
(1, 6)	2	.000416707	0.1	0.1	0.1	.0000416707	.0000416707	.0000416707
(1, 7)	2	.000004333	0.9	0.9	0.9	.000003900	.000003900	.000003900
(2)	1	.000958564	0.0	0.0	0.0	0.0	0.0	0.0
(2, 3)	2	.000001921	0.1	0.1	0.1	.0000001921	.0000001921	.0000001921
(2, 4)	2	.000004817	0.9	0.9	0.9	.000004335	.000004335	.000004335
(2, 5)	2	.000416707	0.9	0.9	0.9	.000375036	.000375036	.000375036
(2, 6)	2	.000416707	0.1	0.1	0.1	.0000416707	.0000416707	.0000416707
(2, 7)	2	.000004333	0.5	0.5	0.5	.000002167	.000002167	.000002167
(3)	1	.000958564	0.0	0.0	0.0	0.0	0.0	0.0
(3, 4)	2	.000004817	0.3	0.3	0.3	.000001445	.000001445	.000001445
(3, 5)	2	.000416707	0.1	1.0	0.1	.0000416707	.000416707	.0000416707
(3, 6)	2	.000416707	0.0	0.0	0.0	0.0	0.0	0.0
(3, 7)	2	.000004333	0.0	0.0	0.0	0.0	0.0	0.0

Table F.5. (Continued)

Path number on outage	Number of outage	$P(B_j)$	$P_{LK}(B_j)$			$P(B_j) \times P_{LK}(B_j)$		
			Bus 6	Bus 3	Bus 5	Bus 6	Bus 3	Bus 5
(4)	1	.002403635	0.3	0.3	0.3	.00072108	.00072108	.00072108
(4, 5)	2	.001044909	0.1	0.0	0.1	.0001044909	0.0	.0001044909
(4, 6)	2	.001044909	0.0	0.0	1.0	0.0	0.0	.001044909
(4, 7)	2	.000010865	0.3	0.3	0.3	.00000326	.00000326	.00000326
(5)	1	.207936810	0.9	0.9	0.9	.18714312	.18714312	.18714312
(5, 6)	2	.090394338	0.0	1.0	1.0	0.0	.090394338	.090394338
(5, 7)	2	.000939945	0.9	0.9	0.9	.000845951	.00084595	.000845951
(6)	1	.207936810	0.0	0.0	0.0	0.0	0.0	0.0
(6, 7)	2	.000939945	0.0	0.0	0.0	0.0	0.0	0.0
(7)	1	.002162185	0.0	0.0	0.0	0.0	0.0	0.0
∴ Probability of system failure at bus k								
$(\sum_{B_j} P(B_j) \times P_{LK}(B_j))$						=	.189808386	.280473264
								.281247628

Table F.6. Reliability study (6 paths)

Path number on outage	Number of outage	$P(B_j)$	$P_{LK}(B_j)$			$P(B_j) \times P_{LK}(B_j)$		
			Bus 6	Bus 3	Bus 5	Bus 6	Bus 3	Bus 5
(none)	0	.480485472	0.0	0.0	0.0	0.0	0.0	0.0
(1)	1	.000962897	0.9	0.9	0.9	.000866607	.000866607	.000866607
(1, 2)	2	.000001930	1.0	1.0	1.0	.000001930	.000001930	.000001930
(1, 3)	2	.00000193	0.5	0.5	1.0	.000000965	.000000965	.000000193
(1, 4)	2	.000004839	0.5	0.0	0.5	.00000242	0.0	.00000242
(1, 5)	2	.000418591	0.9	0.9	0.9	.000376732	.000376732	.000876732
(1, 6)	2	.000418591	0.0	1.0	1.0	0.0	.000418591	.000418591
(2)	1	.000962897	0.5	0.5	0.5	.00048448	.00048448	.00048448
(2, 3)	2	.000001930	0.7	0.0	0.7	.000001351	0.0	.000001351
(2, 4)	2	.000004839	1.0	0.0	1.0	.000004839	0.0	.000004839
(2, 5)	2	.000418591	1.0	1.0	1.0	.00041859	.00041859	.00041859
(2, 6)	2	.000418591	1.0	0.0	0.0	.00041859	0.0	0.0
(3)	1	.000962897	0.0	0.0	0.0	0.0	0.0	0.0
(3, 4)	2	.000004839	0.5	0.5	0.5	.00000242	.00000242	.00000242
(3, 5)	2	.000418591	0.5	1.0	0.5	.000209296	.000418591	.000209296
(3, 6)	2	.000418591	0.3	0.3	0.0	.000125577	.000125577	0.0

Table F.6. (Continued)

Path number on outage	Number of outage	$P(B_j)$	$P_{LK}(B_j)$			$P(B_j) \times P_{LK}(B_j)$		
			Bus 6	Bus 3	Bus 5	Bus 6	Bus 3	Bus 5
(4)	1	.002414500	0.3	0.3	0.3	.00072435	.00072435	.00072435
(4, 5)	2	.001049632	0.5	0.0	0.5	.000524816	0.0	.000524816
(4, 6)	2	.001049632	0.0	0.0	1.0	0.0	0.0	.001049632
(5)	1	.208876755	0.9	0.9	0.9	.18798908	.18798908	.18798908
(5, 6)	2	.090802951	0.0	1.0	1.0	0.0	.090802951	.090802951
(6)	1	.208876755	0.0	0.0	0.0	0.0	0.0	0.0

∴ Probability of system failure at bus k

$$\sum_{B_j} P(B_j) \cdot P_{LK}(B_j) = \underline{\underline{.192152045 \quad .282398177 \quad .283880021}}$$

XVIII. APPENDIX G: PERIODIC ASSIGNMENT OF FIXED CHARGES

In Appendix A, the phantom customer method was applied to a single system with five types of loads. To apply the method to a power pool with a certain number of pool members, one can think of the pool members being represented by customer A, customer B, and so on. But the typical load duration curve of the whole pool would usually be different than that of the individual pool members. We assume that each pool member consumes the "pool power" for 24 hours every day and each member's load should contribute to the pool peak.

The data needed for the study are:

- 1) The load in each member's area during the pool peak load level.
- 2) The load in each area during the pool average load level.
- 3) All data needed for load flow studies including an economic generation schedule for both peak and average loads.

Again, the 6 bus system is studied at both the average load level and peak load level. The loads, the generations, and the power flow in each line are shown in Figures C.1 and C.2. The power flow in each line is summarized in Table G.1 for the average load level and the peak load level.

From the phantom consumer method, it is suggested that Areas 1 and 2 pay an energy charge proportional to the power flows at the average load level and the phantom customer pays the demand charge proportional to the differences of power flows at the load levels. The negative value means that the power flow in the line connecting bus 5 and bus 6

Table G.1. Power flows in the system

Line bus-bus	<u>System load level</u>		Difference of flows at two load levels
	Average	Peak	
1 - 4	13.38 MW	23.9 MW	10.52 MW
1 - 6	11.73 MW	21.0 MW	9.27 MW
2 - 3	7.48 MW	9.53 MW	2.05 MW
2 - 5	12.11 MW	14.67 MW	2.56 MW
3 - 4	10.40 MW	18.59 MW	8.19 MW
4 - 6	2.78 MW	4.86 MW	2.08 MW
5 - 6	1.91 MW	0.34 MW	-1.57 MW

at the peak load level is less than the one at the average load level.

In that case the phantom customer does not exist for that line.

It is assumed arbitrarily that the energy charge should be allocated to Areas 1 and 2 in proportion to the area power consumed at the average load level. Therefore, to comply with this assumption the average load level of a system should be chosen from the load conditions that occur most of the time.

The approaches which were used to construct Tables C.7 and C.9 are used again and the results (ac model) are summarized in Table G.2.

A negative value means that the power flow after the change is less in magnitude than the flow in the same line when the system is operating at the average load level.

Table G.2. Increment of line flows (MW). A. The load in Area 2 is increased from the average load level to the peak load level while the total load in Area 1 is not changed. B. The load in Area 1 is increased from the average load level to the peak load level while the load in Area 2 is not changed

Line flow	A MW	B MW
ΔP_{1-4}	4.17	6.30
ΔP_{1-6}	0.99	8.35
ΔP_{2-3}	2.86	-1.41
ΔP_{2-5}	1.80	0.82
ΔP_{3-4}	6.74	1.43
ΔP_{4-6}	-2.63	5.47
ΔP_{5-6}	1.68	0.08

The phantom customer method utilizes the excess demand on peak to allocate the phantom demand charge because the excess demand on peak creates the phantom customer. In the same line of reasoning one may use the results shown in Table G.2 to allocate the phantom demand charge to Areas 1 and 2.

At the average load level of the system in Figure C.1:

$$\begin{aligned}\text{Total system load} &= 16.25 + 9.75 + 17.9 \\ &= 43.9 \text{ MW}\end{aligned}$$

$$\begin{aligned}\text{Total load of Area 1} &= 16.25 + 9.75 = 26.00 \text{ MW} \\ &= 59.23\% \text{ of system load}\end{aligned}$$

$$\begin{aligned}\text{Total load of Area 2} &= 17.9 \text{ MW} \\ &= 40.77\% \text{ of system load.}\end{aligned}$$

Table G.3. Cost allocation by the phantom customer method

Line Bus-bus	<u>Energy charge in MW</u>			Phantom demand charge MW	<u>Phantom demand charge allocated to</u>	
	Total	Area 1	Area 2		Area 1	Area 2
1 - 4	13.38 MW	7.31 MW	6.07 MW	10.52	6.34 MW	4.18 MW
1 - 6	11.73	6.95	4.78	9.27	8.30	0.97
2 - 3	7.48	4.43	3.05	2.05	0.00	2.05
2 - 5	12.11	7.18	4.93	2.56	0.80	1.76
3 - 4	10.40	6.16	4.24	8.19	1.43	6.74
4 - 6	2.78	1.35	0.93	2.08	2.08	0.00
5 - 6	1.91	1.13	0.78	0.00	0.00	0.00

Notice in Table G.2 that a negative increment indicates that the area in which the load changed causes less flow in the line at the system peak level than the flow at the system average load level. In this case the area incremented should pay no part of the phantom costs.

Sample of calculation: Line 1 - 4

Total energy charge in MW	= 13.38
Allocated to Area 1	= $59.23/100 \times 13.38 = 7.31$ MW
Allocated to Area 2	= $40.77/100 \times 13.38 = 6.07$ MW
Total phantom demand charge	= 10.52 MW
Allocated to Area 1	= $6.30/10.47 \times 10.52 = 6.34$ MW
Allocated to Area 2	= $4.17/10.47 \times 10.52 = 4.18$ MW
Total charge to Area 1	= $7.31 + 6.34 = 13.65$ MW
Total charge to Area 2	= $6.07 + 4.18 = 10.25$ MW

Table G.4. Summary of results in MW and in percent of fixed charges

Line bus-bus	Total charge to Area 1		Total charge to Area 2	
	MW	%	MW	%
1 - 4	13.65	58.0	10.24	24.0
1 - 6	15.25	72.5	5.75	27.5
2 - 3	4.43	46.6	5.10	53.4
2 - 5	7.97	53.5	6.70	46.5
3 - 4	7.59	41.0	10.98	59.0
4 - 6	3.43	79.0	0.93	21.0
5 - 6	1.13	59.2	0.78	40.8